# Synthetic Controls with Staggered Adoption

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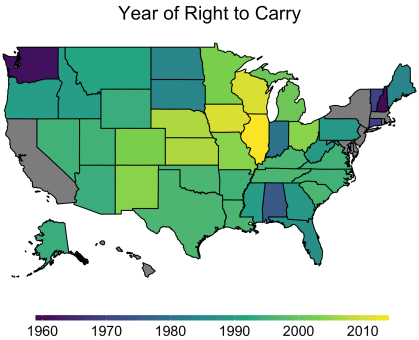
(joint work with Avi Feller and Jesse Rothstein)

ENAR 2021 Spring Meeting

March 2021

# What is the impact of right-to-carry laws on violent crime?

- *1959 – 2014:* 42 states enact right-to-carry



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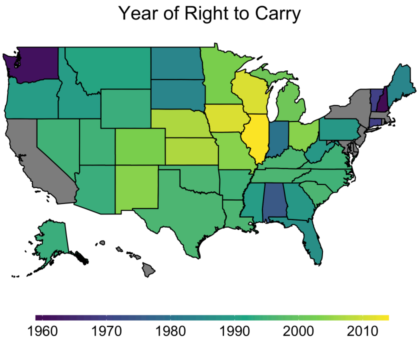
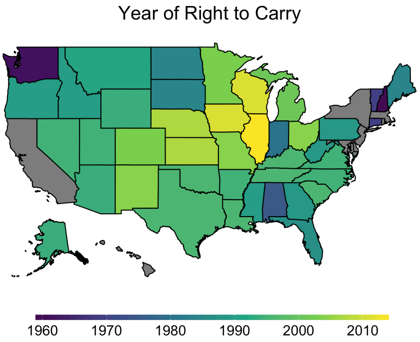
- "More guns, less crime"?
  [Lott and Mustard, 1997]



# What is the impact of right-to-carry laws on violent crime?

- *1959 – 2014:* 42 states enact right-to-carry
- "More guns, less crime"?
  [Lott and Mustard, 1997]
- New research says no
  [Donohue et al., 2019]



## Estimating effects under staggered adoption

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*Partially pooled SCM*

- Modify optimization problem to target overall and state-specific fit
- Account for level differences with Intercept-Shifted SCM

# What do we want to estimate?

Units: $i = 1, \ldots, N$, $J$ total treated units

Time: $t = 1, \ldots, T$, treatment times $T_1, \ldots, T_J, \infty$

Outcome: at *event time* $k$, $Y_{i,T_j+k}$

- Some assumptions to write down potential outcomes
  [Athey and Imbens, 2018; Imai and Kim, 2019]

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Basic building block:

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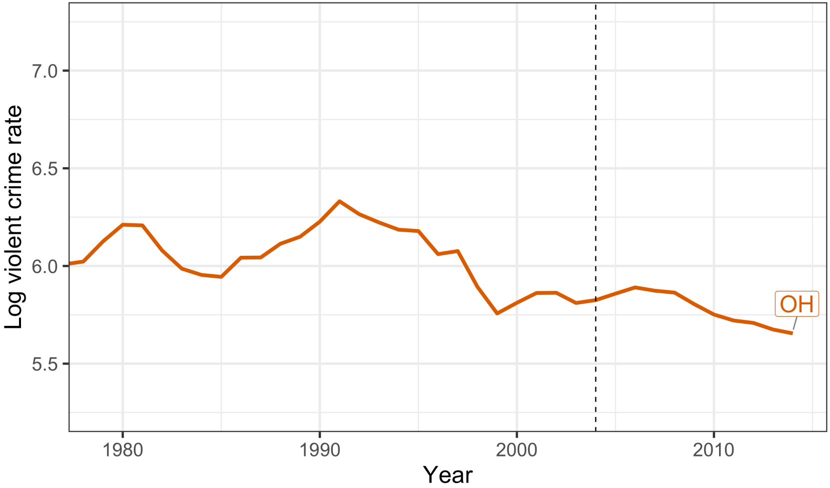
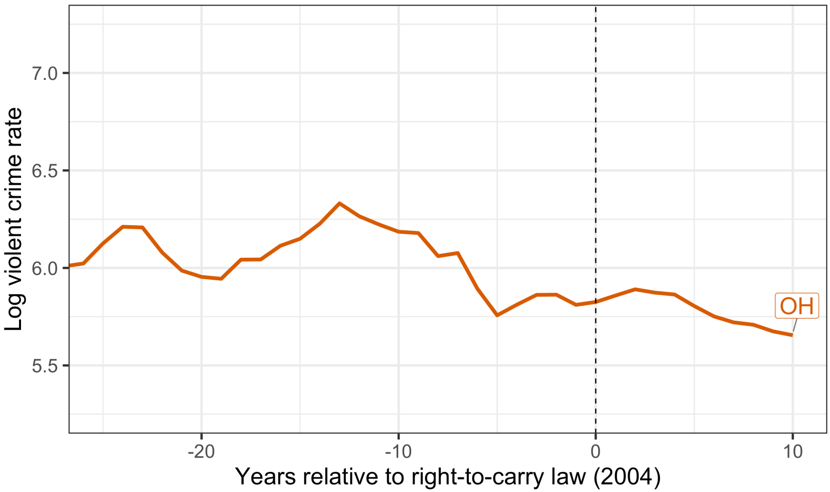
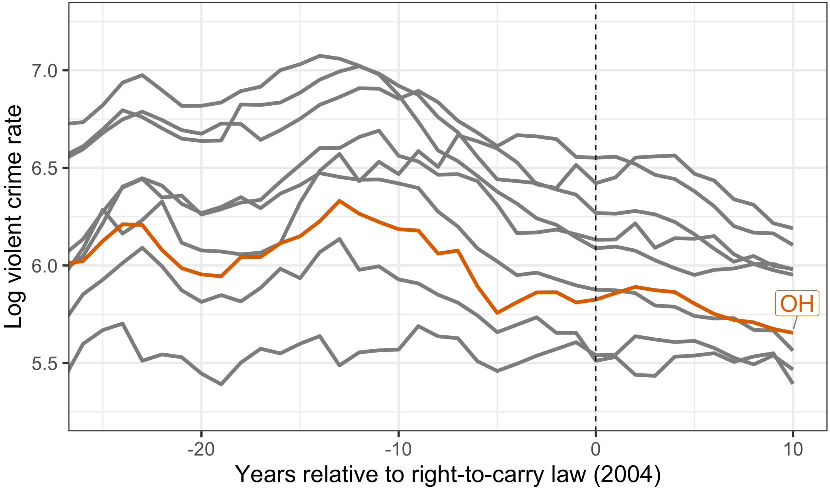
Average at event time $k$:

$$\text{ATT}_k = \frac{1}{J} \sum_{j=1}^{J} \tau_{jk}$$

# Separate
# Synthetic Controls


Log violent crime rate
7.0
6.5
6.0
5.5
1980
1990
2000
2010
Year
OH


Log violent crime rate
7.0
6.5
6.0
5.5
OH
-20
-10
0
10
Years relative to right-to-carry law (2004)


Log violent crime rate
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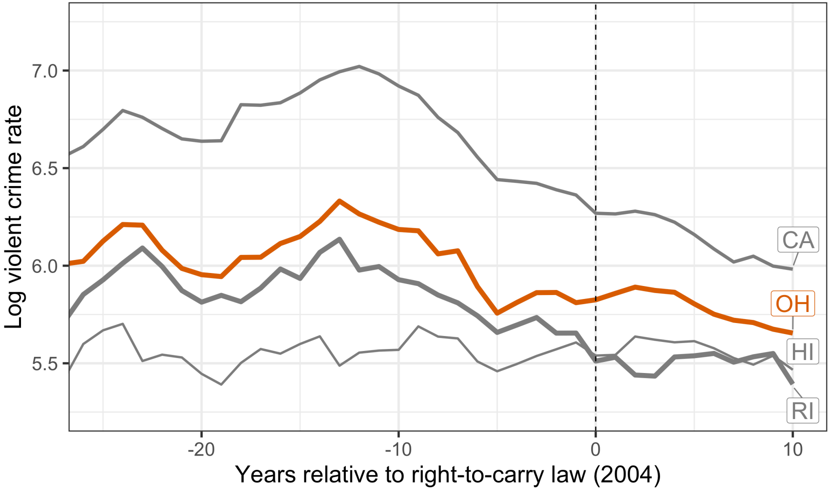
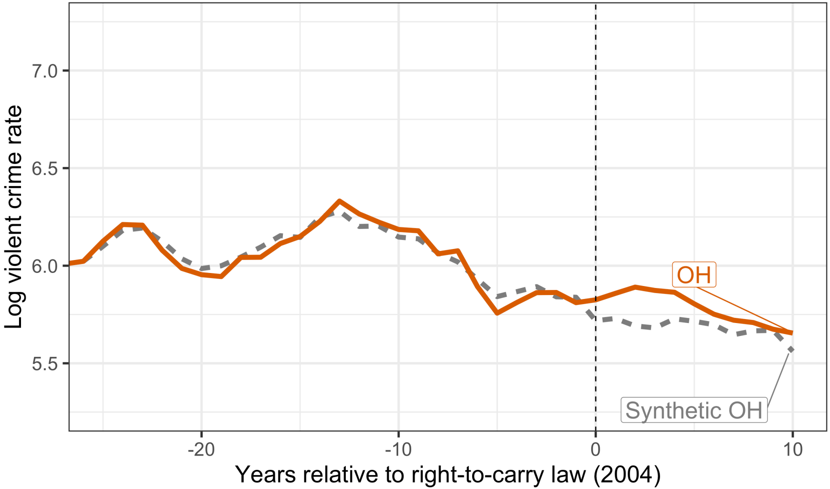
OH




$$\min_{\gamma \in \Delta^{\text{scm}}} \left\| Y_{\text{OH}\ell} - \sum_{i \neq \text{OH}} \gamma_i Y_{i\ell} \right\|_2^2 + \text{penalty}$$

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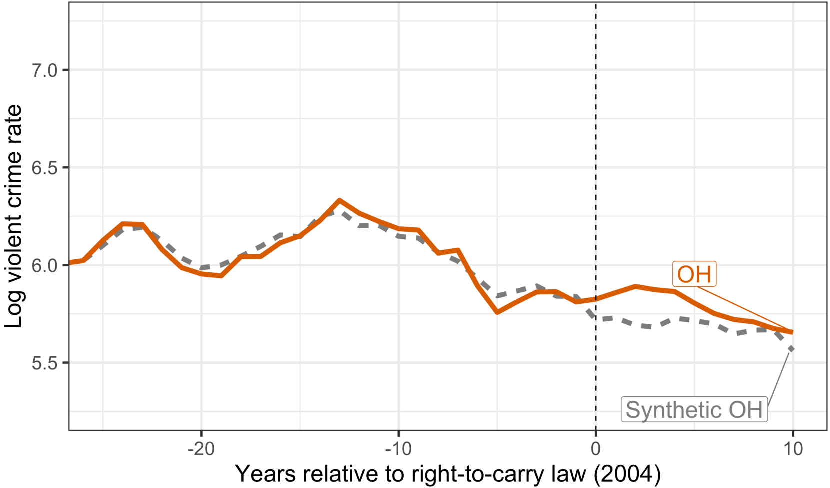
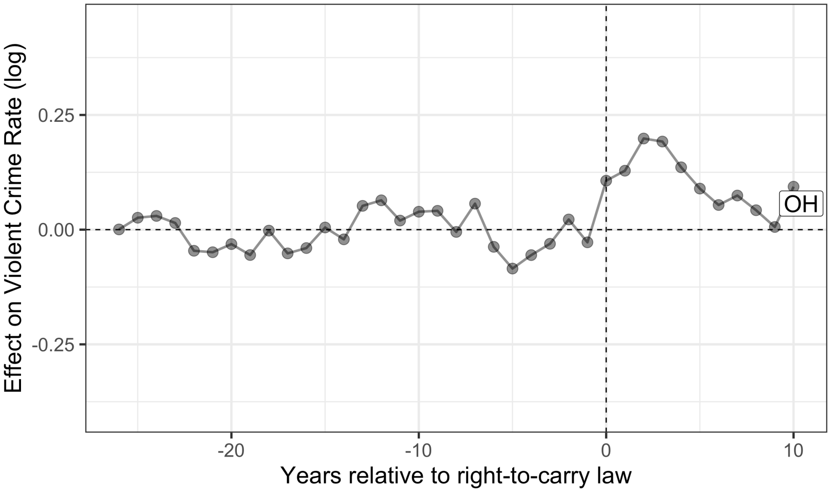



$$\min_{\gamma \in \Delta^{\text{scm}}} \quad \|\text{State Balance}\|_2^2 + \text{penalty}$$

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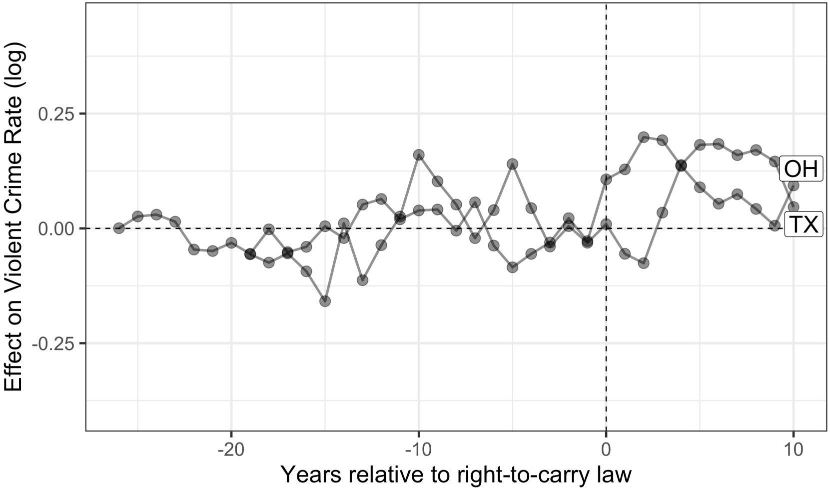
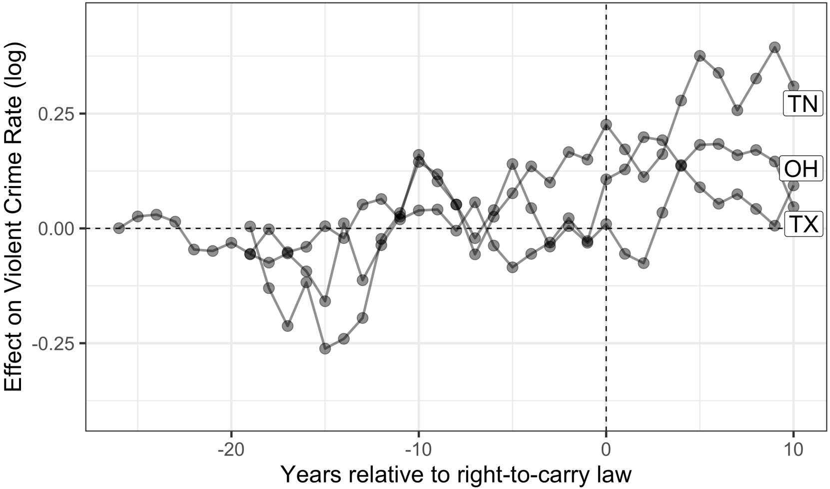



$$\min_{\Gamma \in \Delta^{\text{scm}}} \frac{1}{J} \sum_{j=1}^{J} \left\| \text{State Balance}_j \right\|_2^2 + \text{penalty}$$

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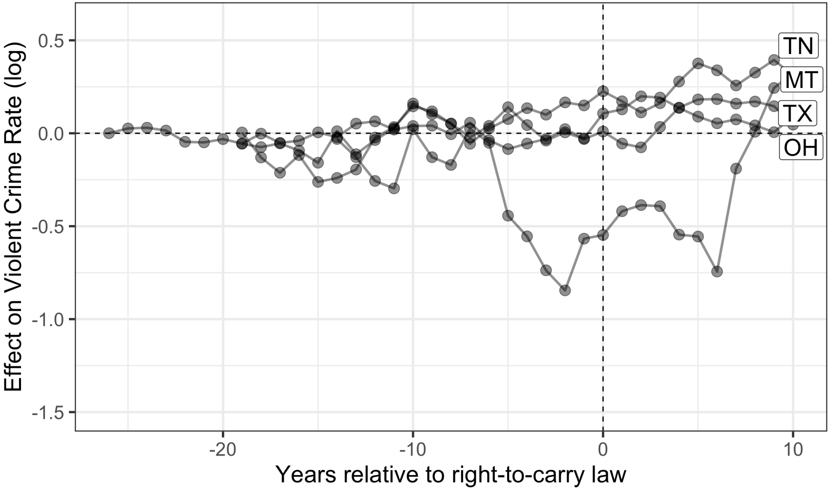


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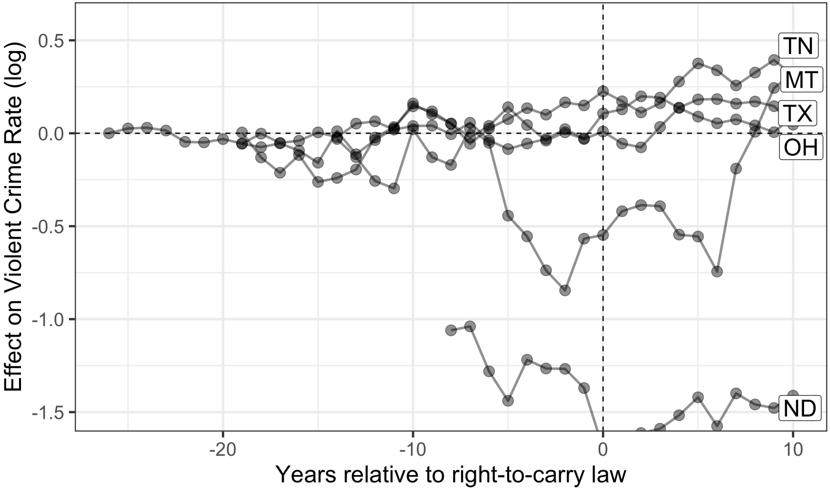


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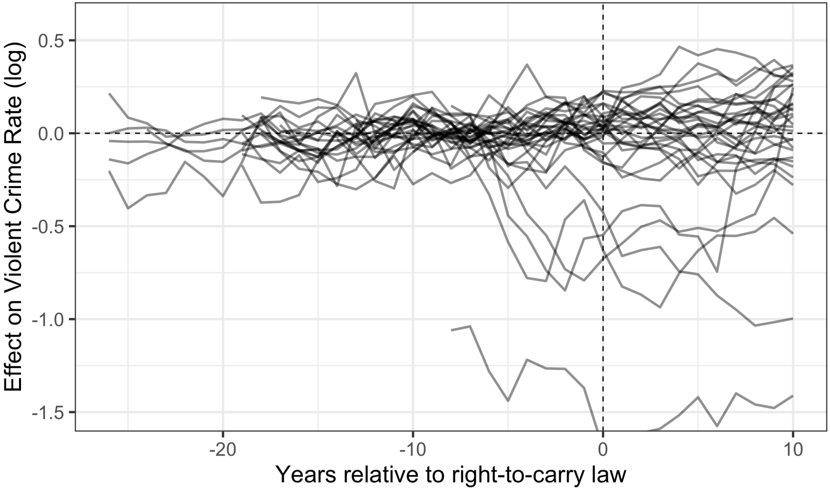


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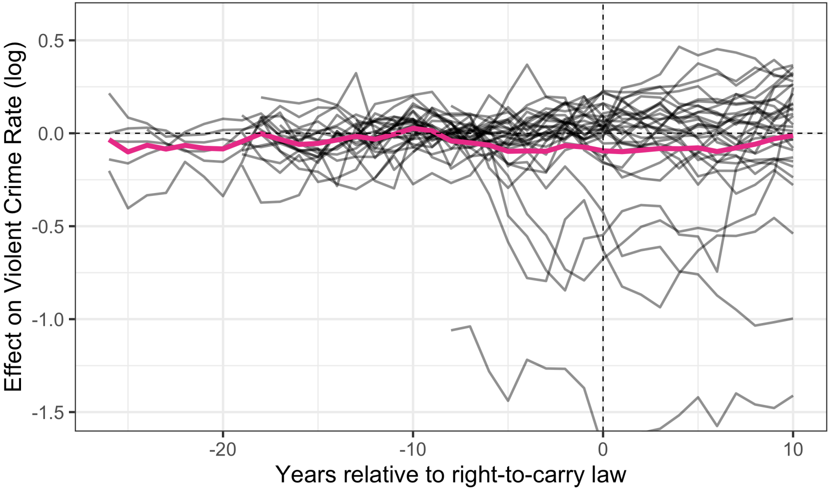


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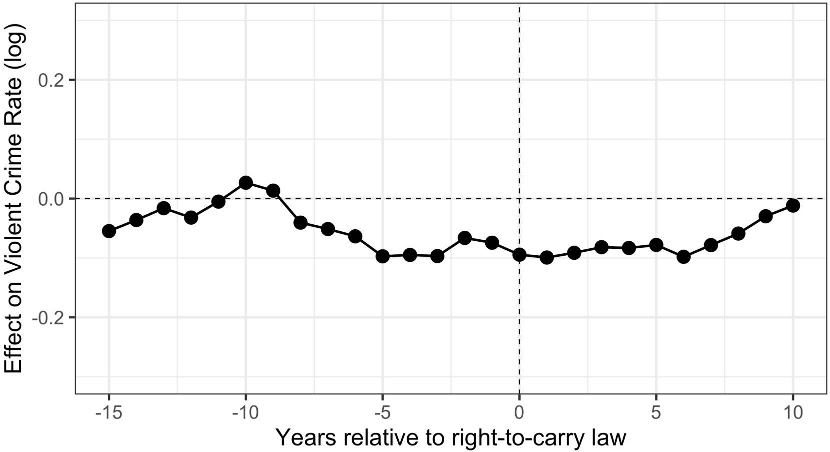
## Separate SCM

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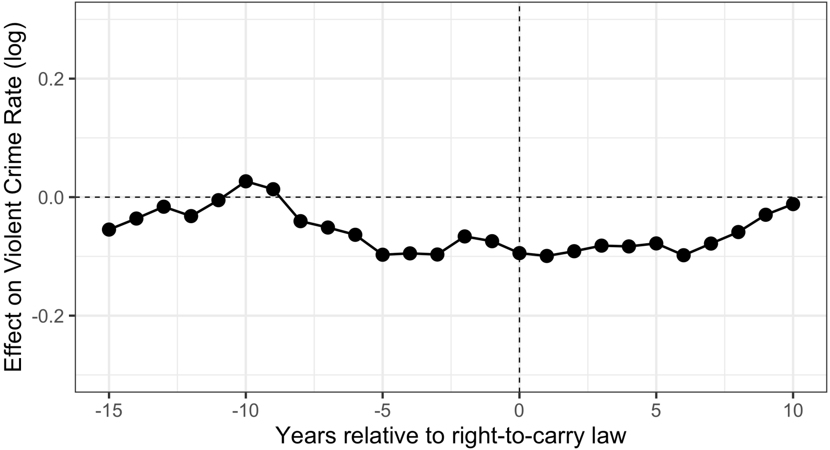
# Partially Pooled SCM

## Separate SCM

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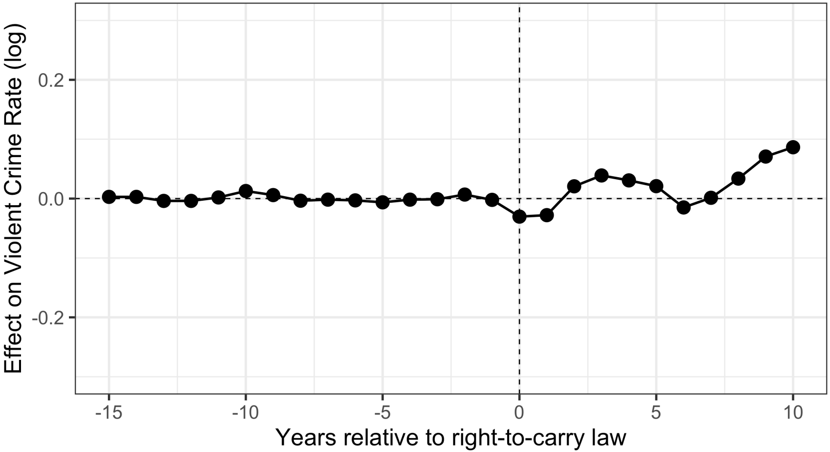
## Pooled SCM

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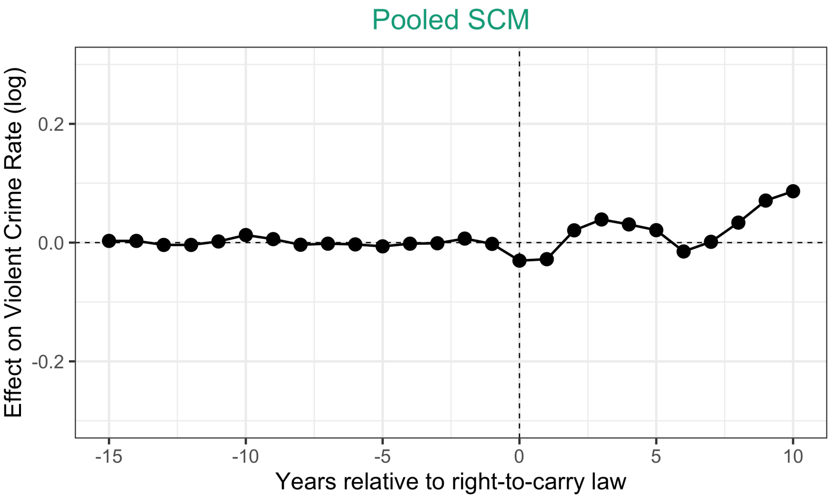
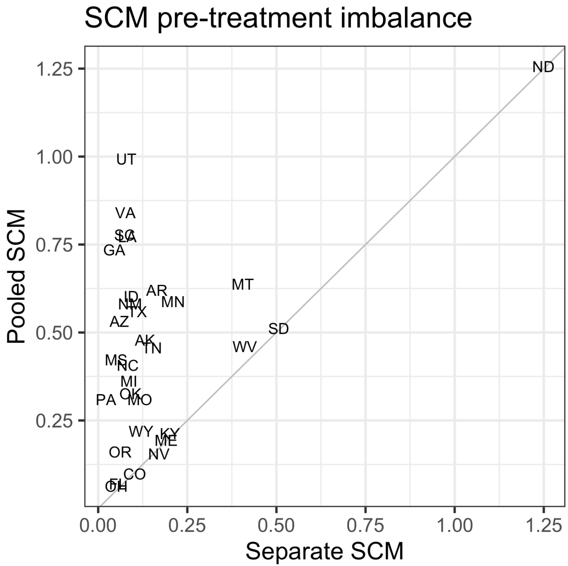
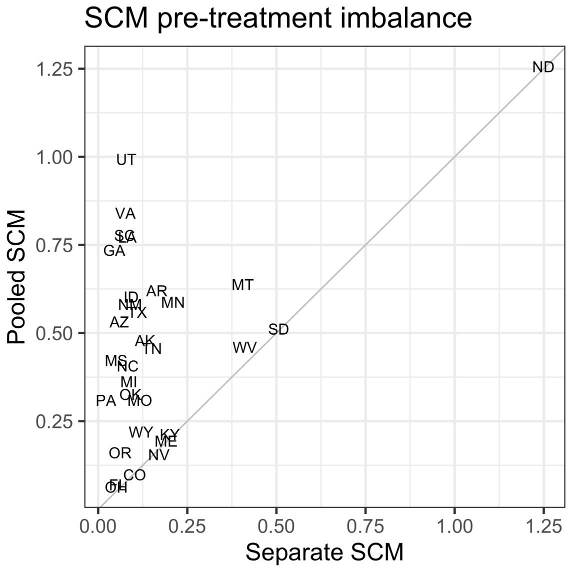



$$\min_{\Gamma \in \Delta^{\text{scm}}} \|\text{Pooled Balance}\|_2^2 + \text{penalty}$$




Pooled Balance is better!

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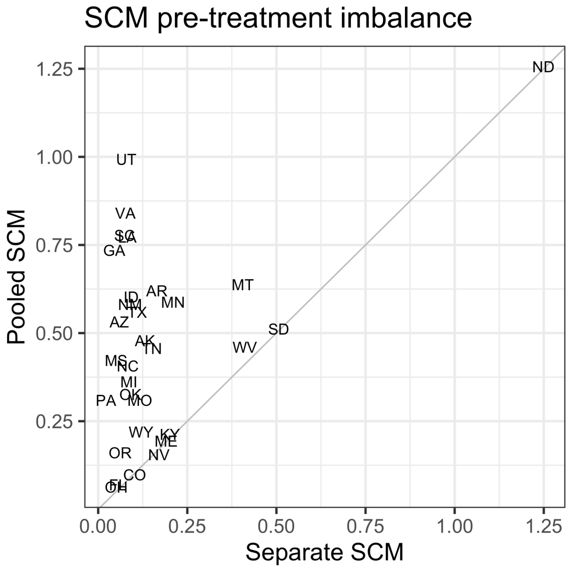
... but State Balance is worse

- Bad for **state estimates**

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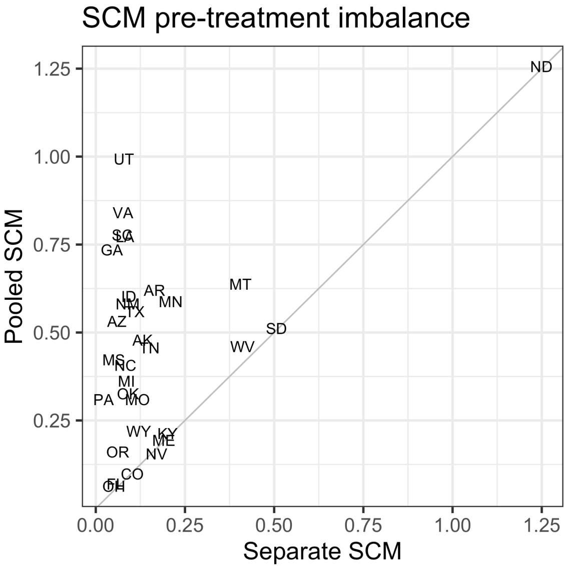
- When DGP varies over time

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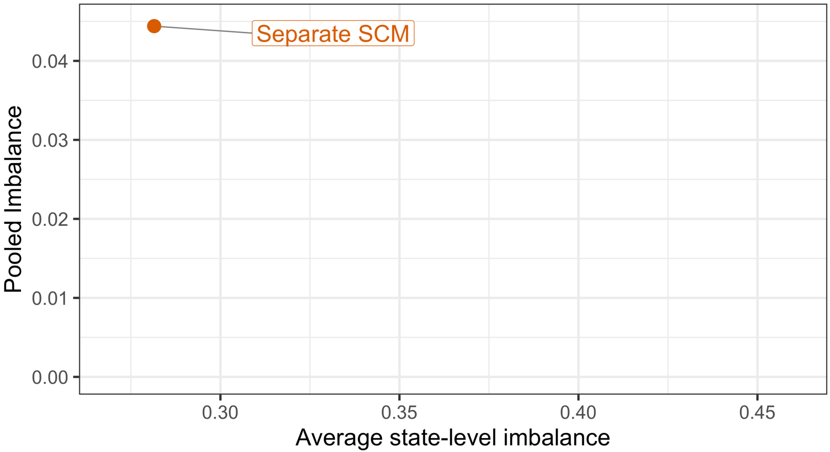
- When DGP varies over time

Find weights that balance both Pooled Balance and State Balance

## Balance possibility frontier




$$\min_{\Gamma \in \Delta^{\text{scm}}} \quad \nu \left\| \text{Pooled Balance} \right\|_2^2 + \frac{1-\nu}{J} \sum_{j=1}^{J} \left\| \text{State Balance}_j \right\|_2^2 + \text{penalty}$$

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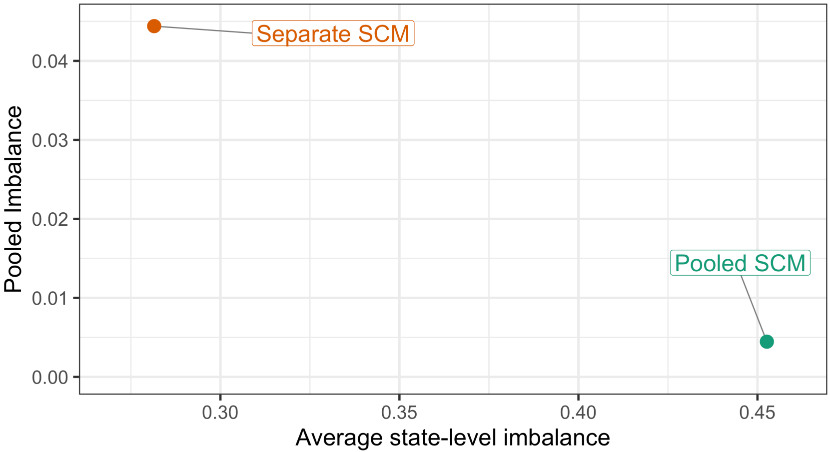


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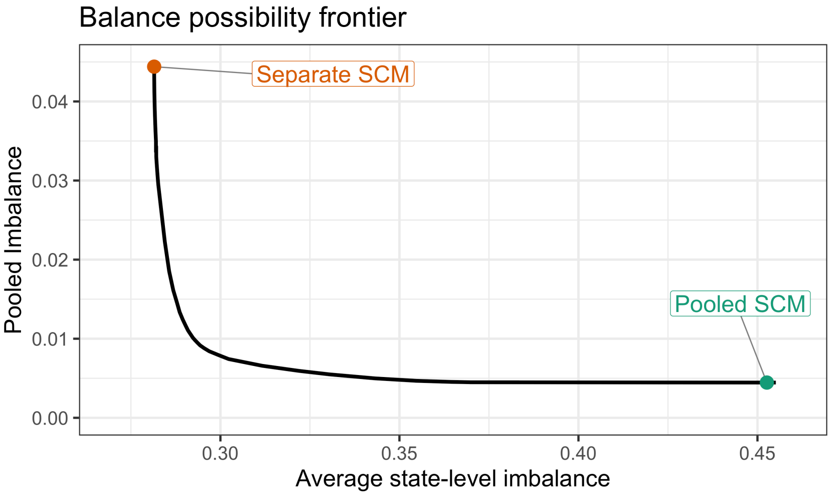


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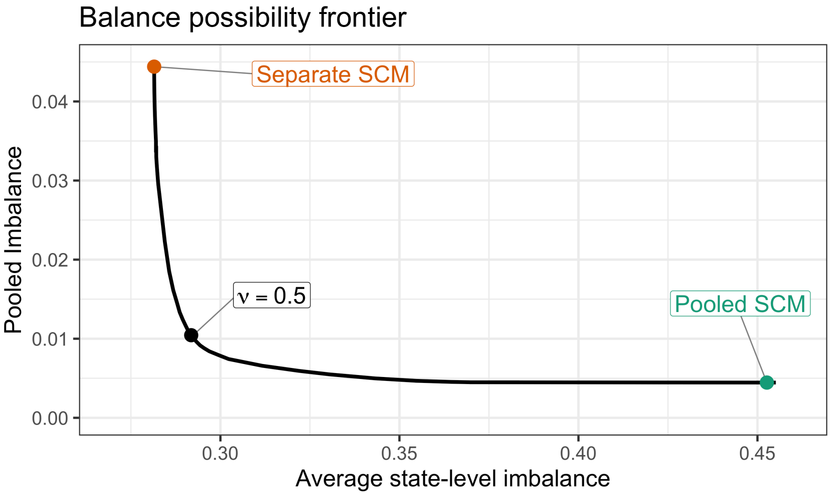


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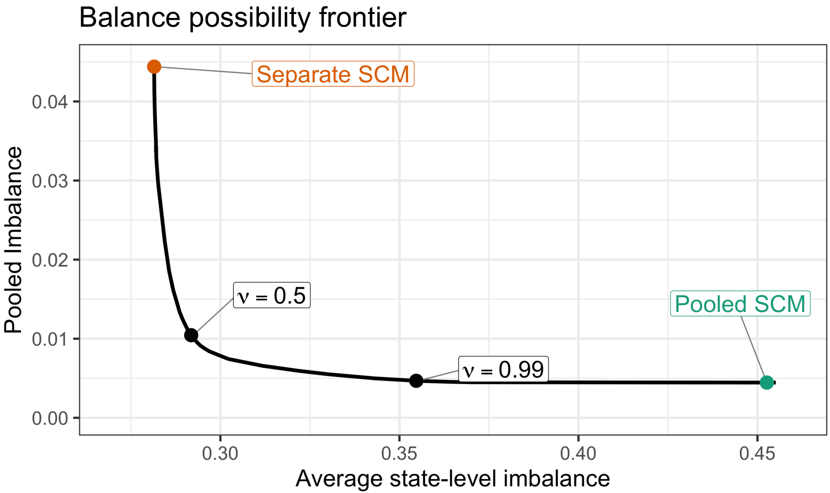


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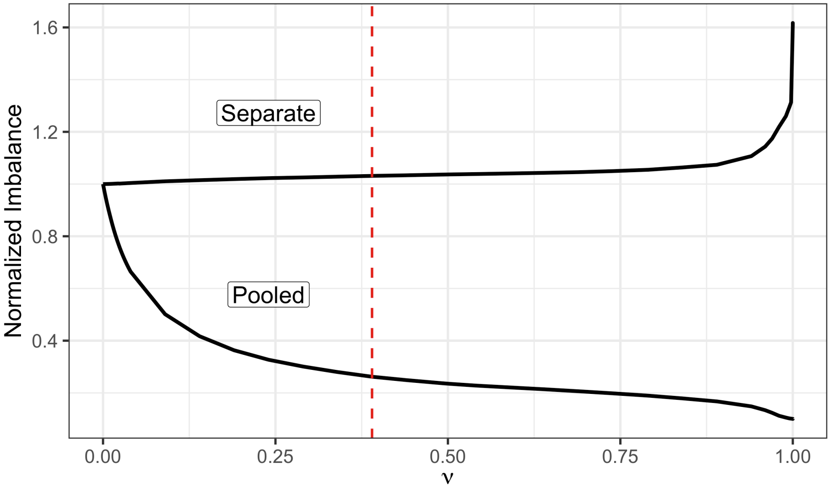
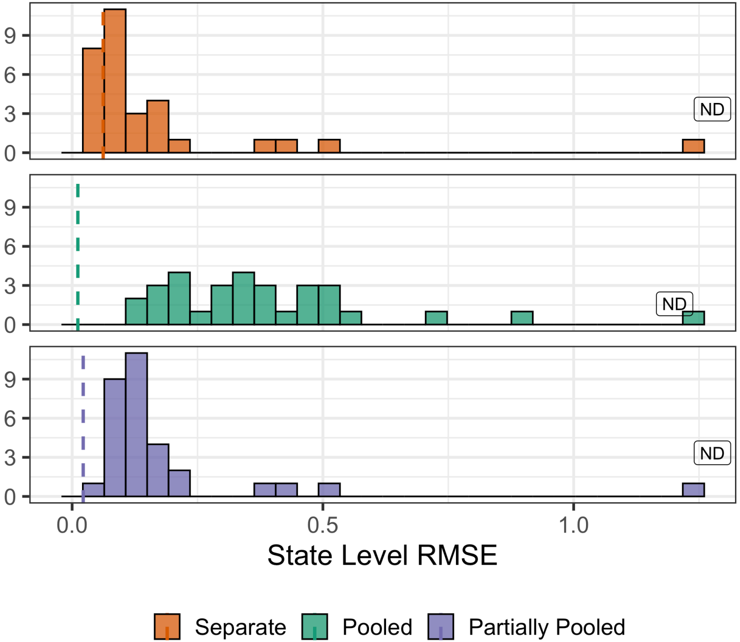
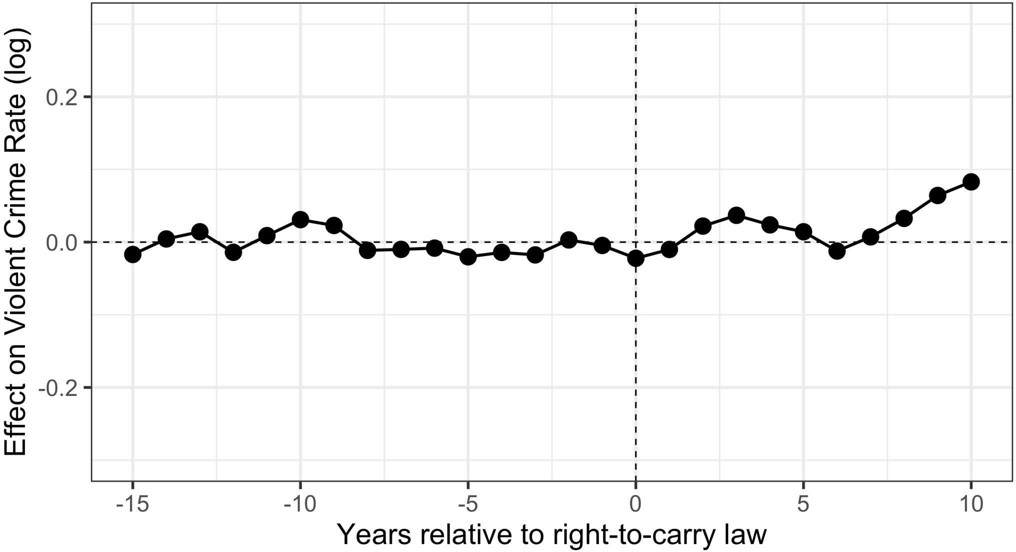



$$\text{Heuristic for } \nu = \frac{\|\text{Pooled Balance}\|_2}{\frac{1}{\sqrt{J}}\sum_{j=1}^{J}\|\text{State Balance}_j\|_2} \qquad \text{fit with } \nu = 0$$



9
6
3
0
ND
0.0
0.5
1.0
State Level RMSE
Separate
Pooled
Partially Pooled

## Partially Pooled SCM



# Extensions

# Intercept-Shifted SCM

Adjust for level differences by adding an intercept to the optimization problem

[Doudchenko and Imbens, 2017; Ferman and Pinto, 2018]

$$\hat{Y}^*_{j,T_j+k}(\infty) = \hat{\alpha}_j + \sum_i \hat{\gamma}^*_{ij} Y_{i,T_j+k}$$

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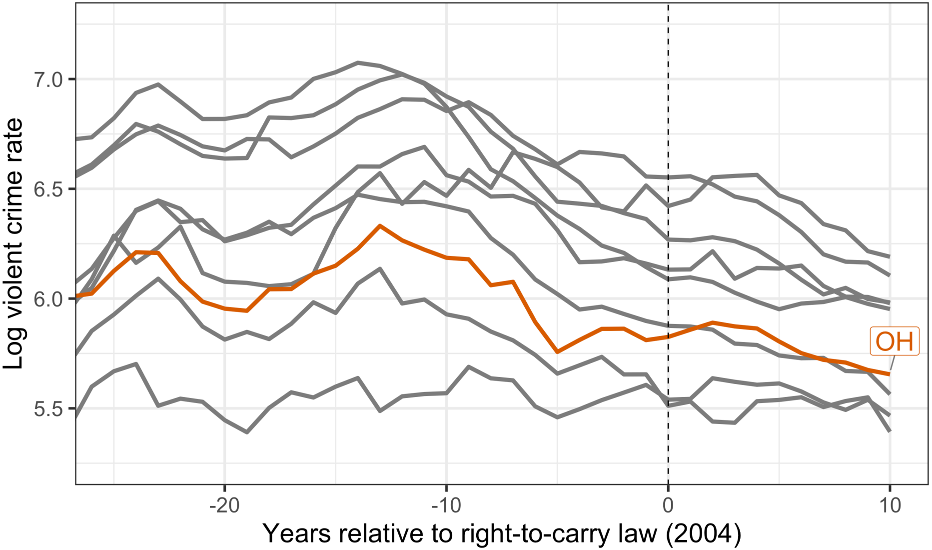
Solution: De-meaning by pre-treatment average $\bar{Y}^{\text{pre}}_{i,T_j}$

Treatment effect estimate is **weighted difference-in-differences**

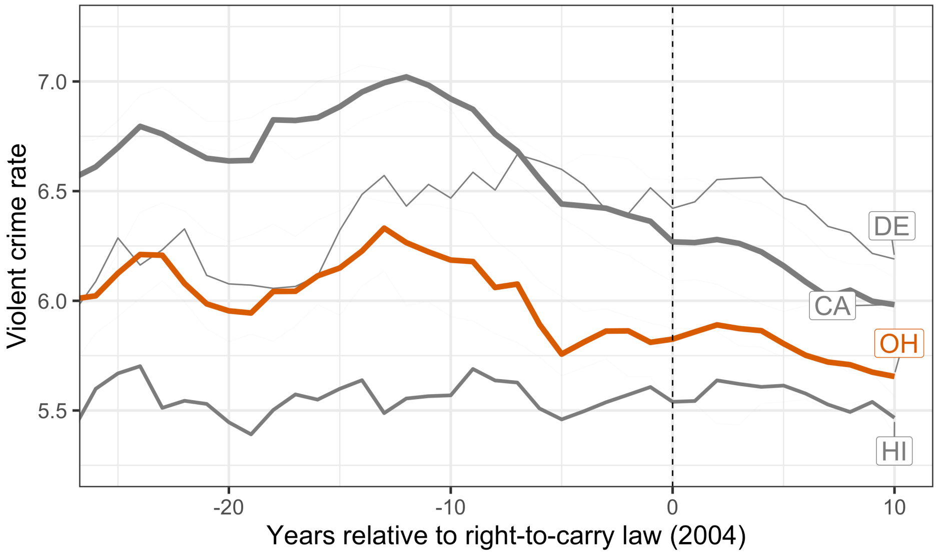
$$\hat{\tau}_{jk} = \left(Y_{j,T_j+k} - \bar{Y}^{\text{pre}}_{j,T_j}\right) - \sum_{i=1}^{N} \hat{\gamma}^*_{ij} \left(Y_{i,T_j+k} - \bar{Y}^{\text{pre}}_{i,T_j}\right)$$

→ Uniform weights recover "stacked" DiD [Abraham and Sun, 2018]

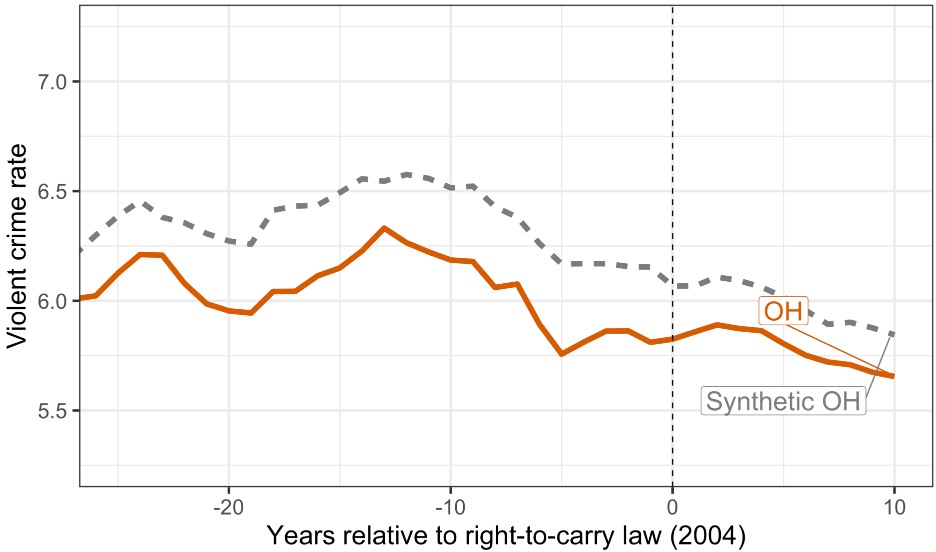
→ Similar in form to P-score weighted DiD [Abadie, 2005; Callaway and Sant'Anna, 2020]

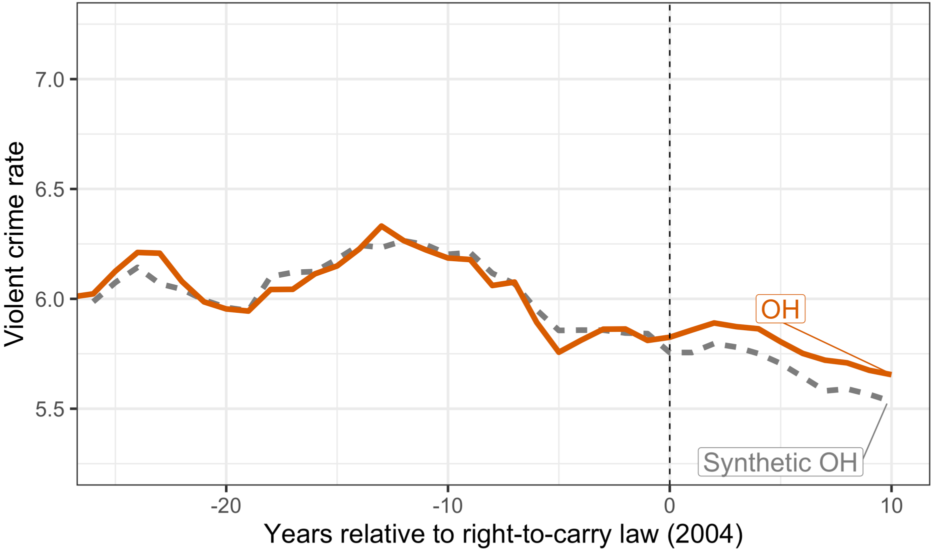


7.0
6.5
6.0
5.5
Log violent crime rate
-20
-10
0
10
Years relative to right-to-carry law (2004)
OH



7.0
6.5
6.0
5.5
-20
-10
0
10
Violent crime rate
Years relative to right-to-carry law (2004)
DE
CA
OH
HI


Violent crime rate
7.0
6.5
6.0
5.5
-20
-10
0
10
Years relative to right-to-carry law (2004)
OH
Synthetic OH


Violent crime rate
7.0
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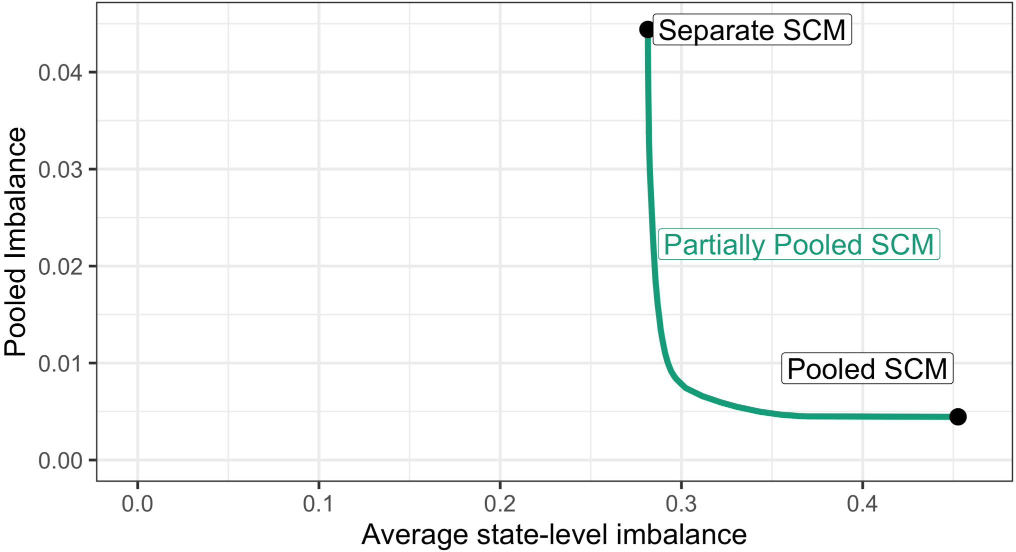
10
Years relative to right-to-carry law (2004)
OH
Synthetic OH

# Balance possibility frontier



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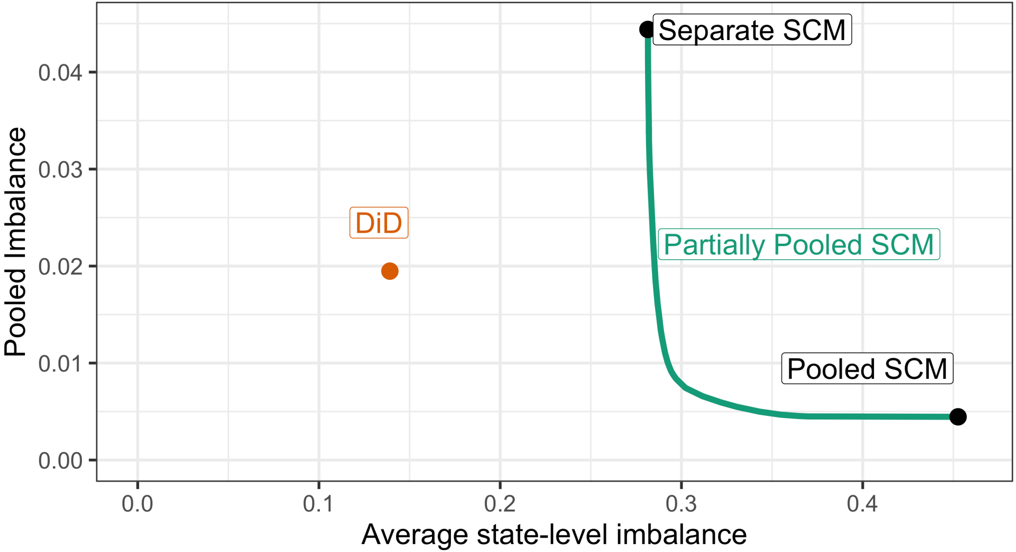

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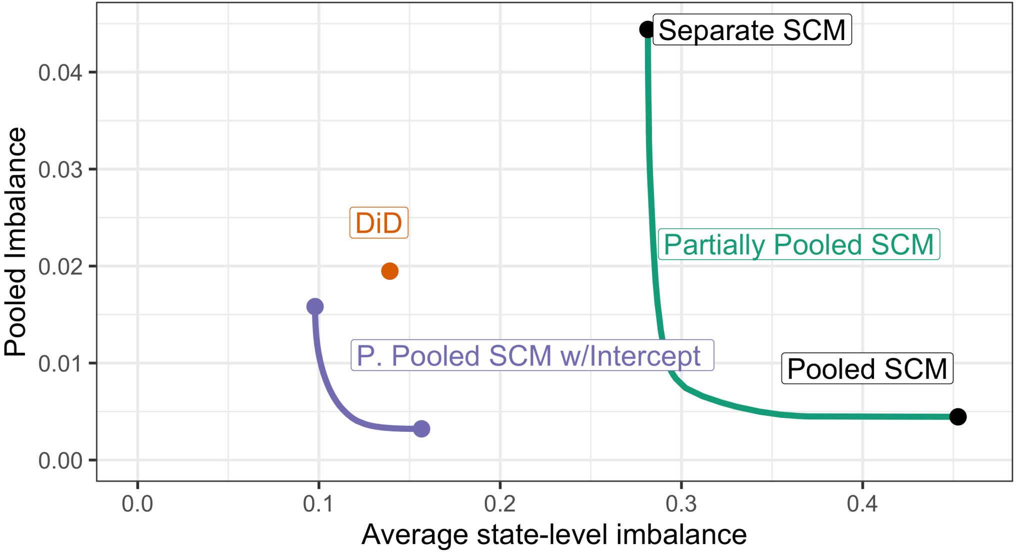
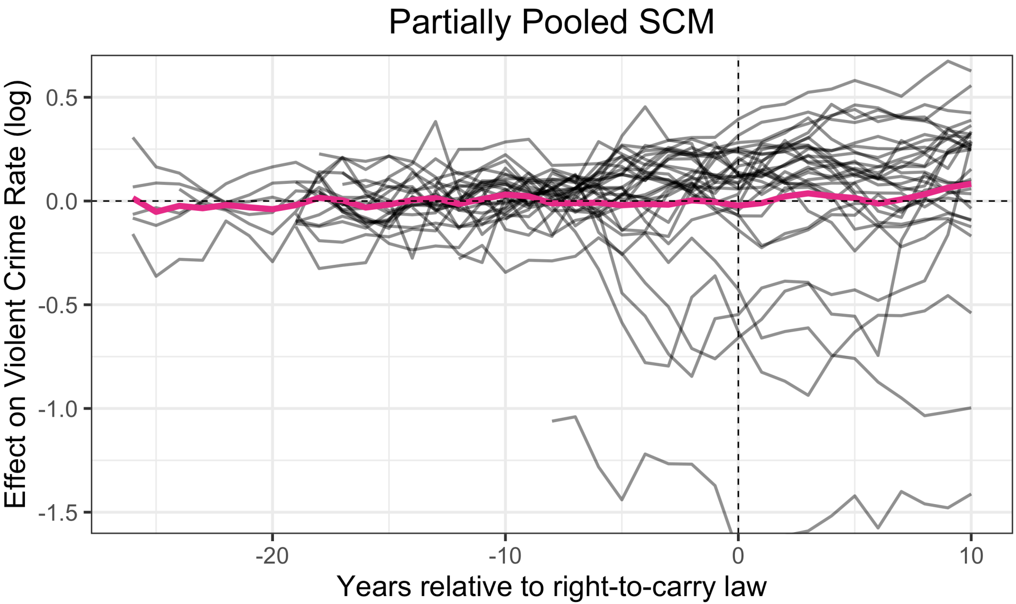
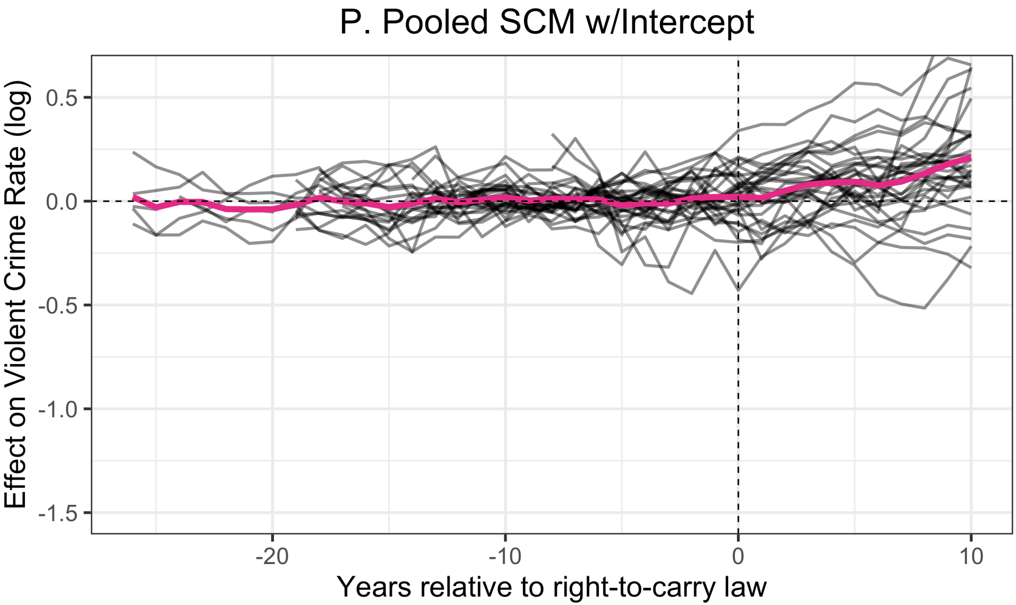



Partially Pooled SCM
Effect on Violent Crime Rate (log)
0.5
0.0
-0.5
-1.0
-1.5
-20
-10
0
10
Years relative to right-to-carry law


P. Pooled SCM w/Intercept
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-10
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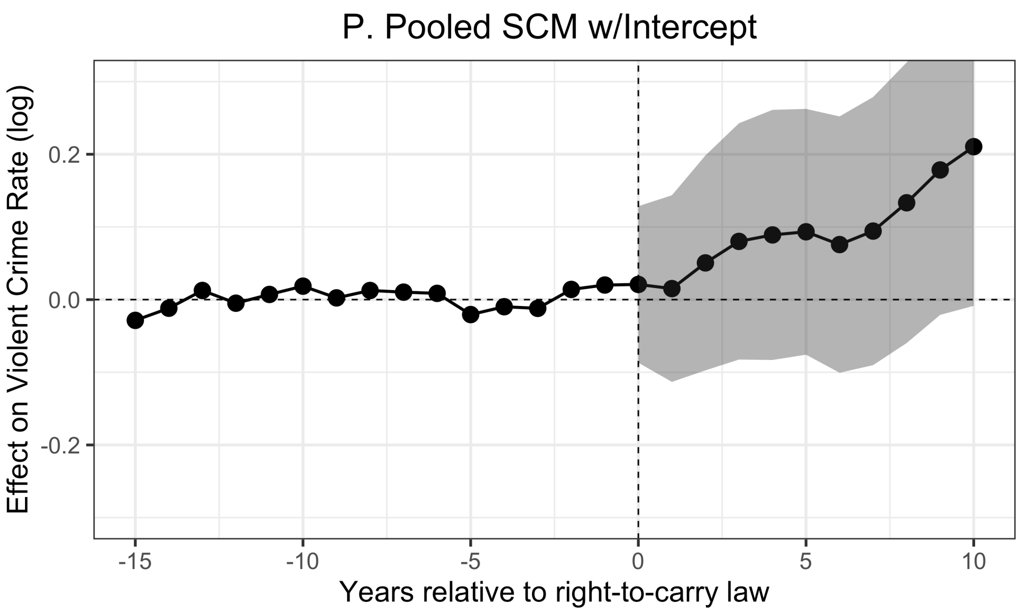

P. Pooled SCM w/Intercept
Effect on Violent Crime Rate (log)
0.2
0.0
-0.2
-15
-10
-5
0
5
10
Years relative to right-to-carry law

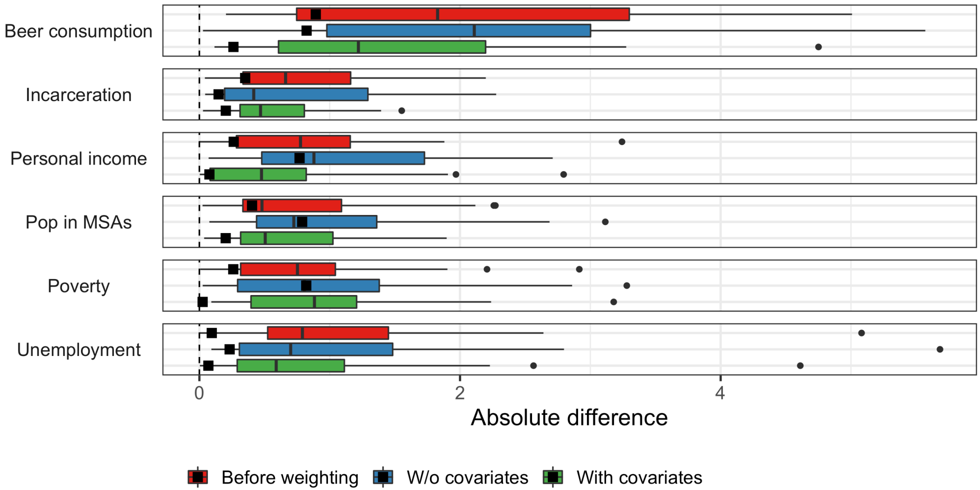
# Incorporating auxiliary covariates

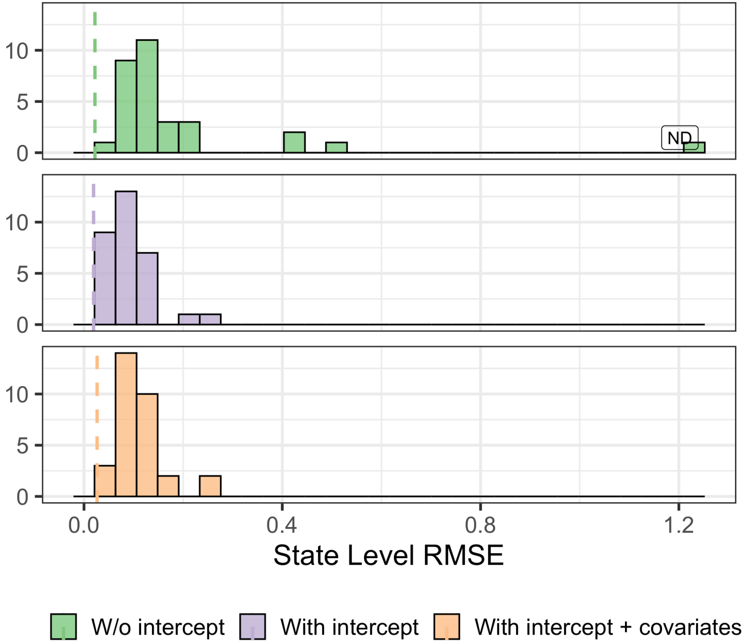
Often have additional covariates other than the main outcome

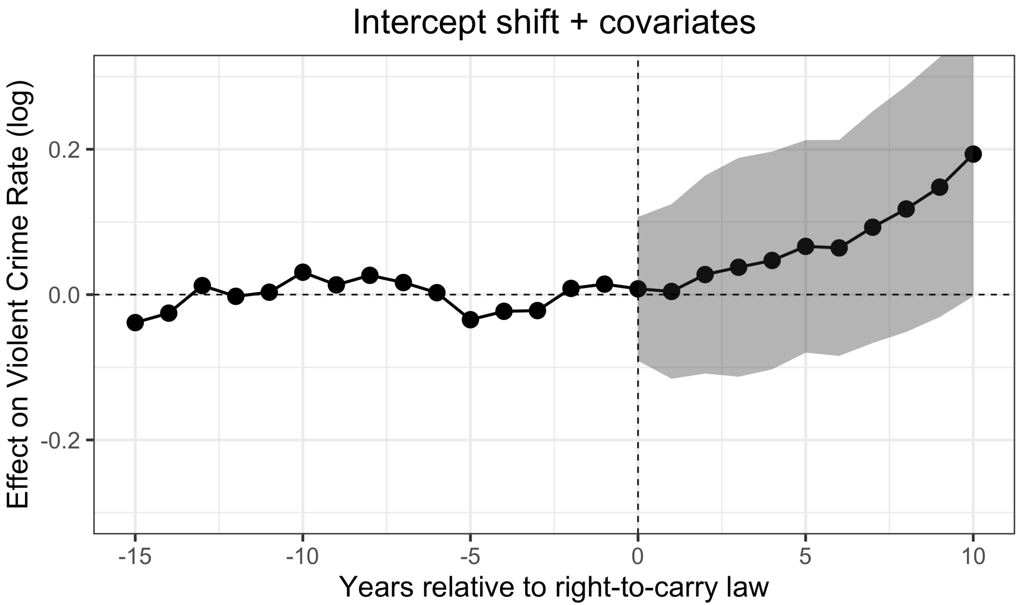
- E.g. poverty, unemployment, incarceration, and police staffing rates
- Demographics

Same trade-off between State Balance and Pooled Balance

We focus on fixed covariates, but time-varying covariates are similar


Beer consumption
Incarceration
Personal income
Pop in MSAs
Poverty
Unemployment
0
2
4
Absolute difference
Before weighting
W/o covariates
With covariates


10
5
0
ND
10
5
0
10
5
0
0.0
0.4
0.8
1.2
State Level RMSE
W/o intercept
With intercept
With intercept + covariates


Intercept shift + covariates
Effect on Violent Crime Rate (log)
0.2
0.0
-0.2
-15
-10
-5
0
5
10
Years relative to right-to-carry law

# Recap

*This paper:* Extend SCM to staggered adoption

- Find weights that control State Balance and Pooled Balance
- Include an intercept to adjust for level differences
- Incorporate auxiliary covariates

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*This paper:* Extend SCM to staggered adoption

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Thank you!

https://arxiv.org/abs/1912.03290

https://github.com/ebenmichael/augsynth

# Appendix

# The role of State Balance and Pooled Balance

## Generalization of parallel trends: Linear Factor Model

$$Y_{it}(\infty) = \phi_i' \mu_t + \varepsilon_{it}$$

## The role of State Balance and Pooled Balance

Generalization of parallel trends: Linear Factor Model

$$Y_{it}(\infty) = \phi_i' \mu_t + \varepsilon_{it}$$

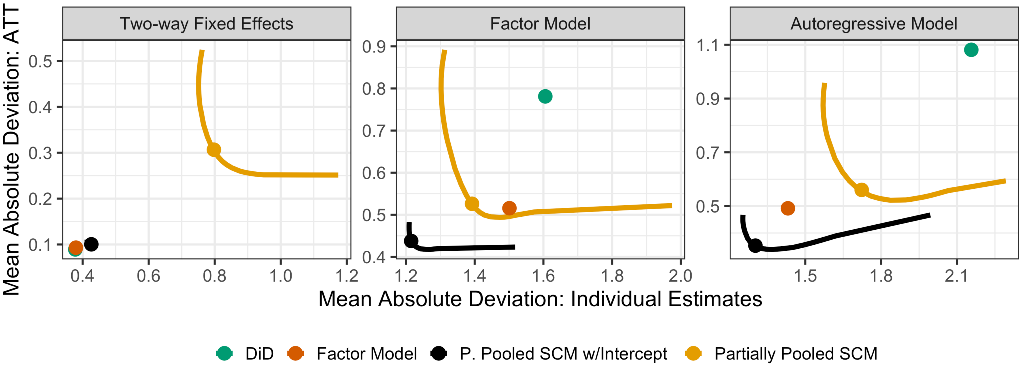
### Error for ATT

$$\left|\widehat{\text{ATT}}_0 - \text{ATT}_0\right| \lesssim \|\bar{\mu}\|_2 \|\text{Pooled Balance}\|_2 + S\sqrt{\sum_{j=1}^{J} \|\text{State Balance}_j\|_2^2} + \sqrt{\frac{\log NJ}{T}}$$

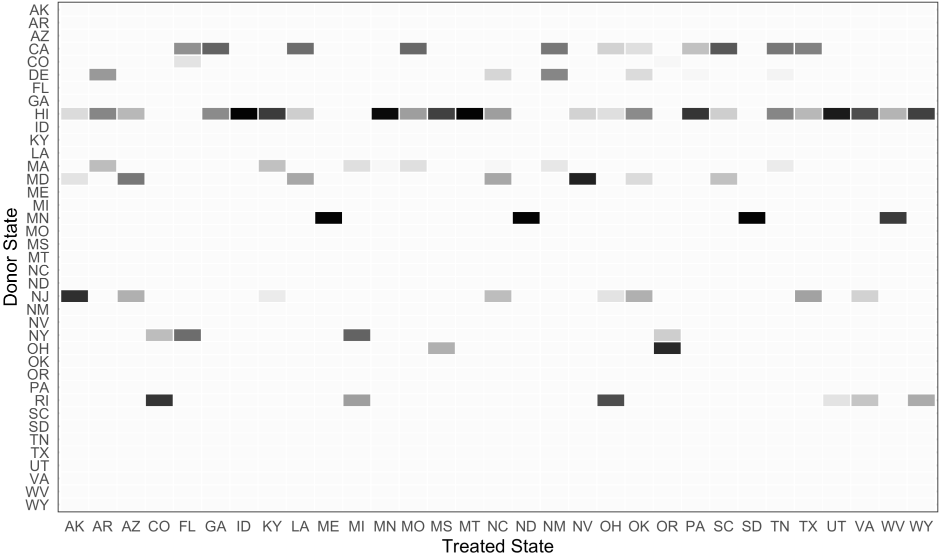
Level of **heterogeneity over time** is important

- $\bar{\mu}$ is the average factor value $\rightarrow$ importance of Pooled Balance
- $S$ is the factor standard deviation $\rightarrow$ importance of State Balance
- Special case: unit fixed effects, only Pooled Balance matters

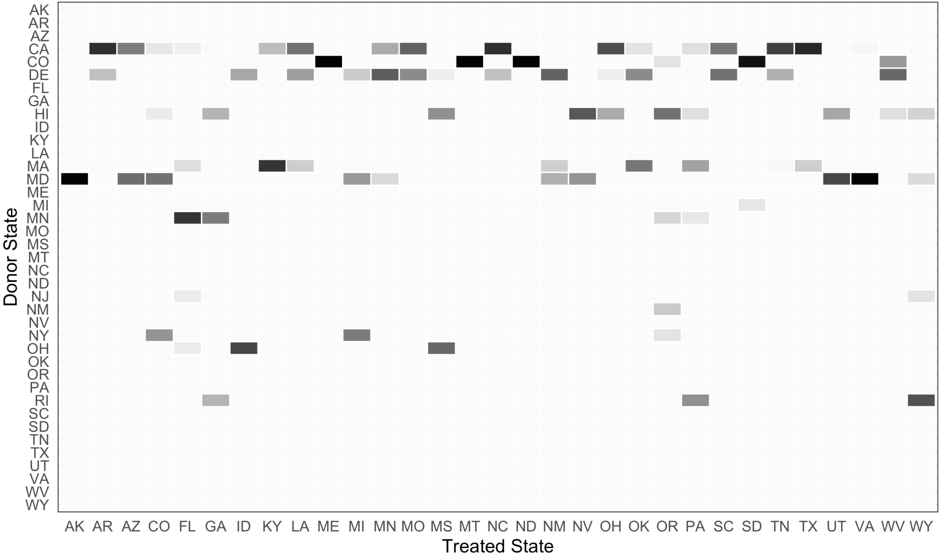
# Simulation study

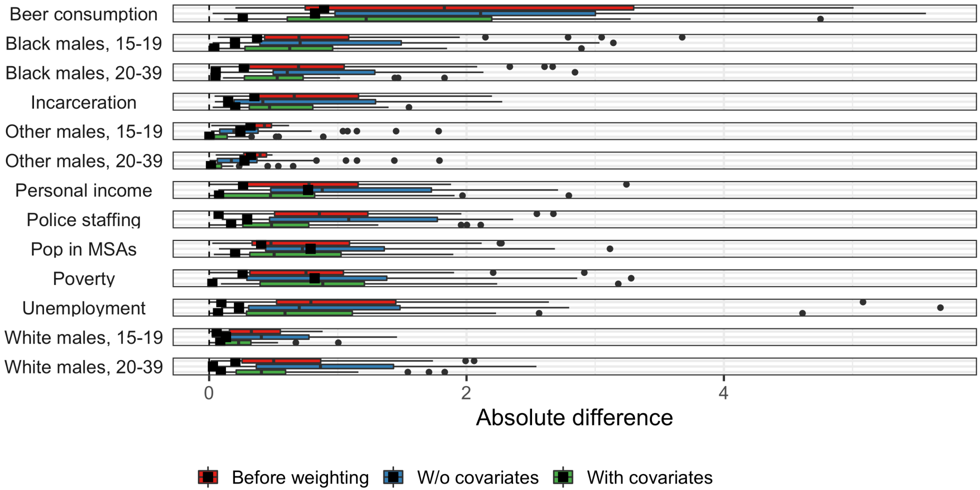


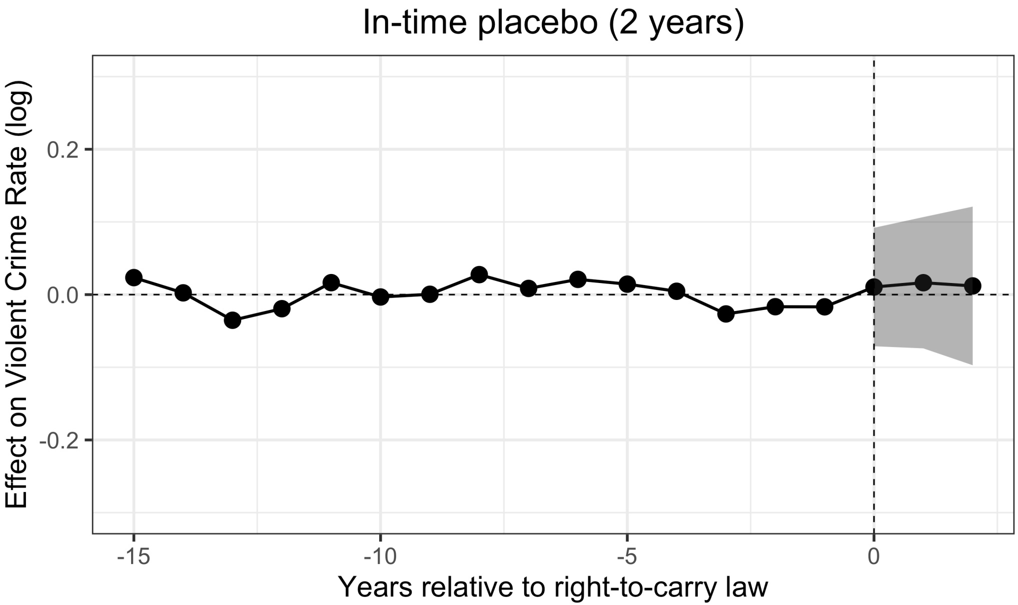
# Partially pooled SCM weights

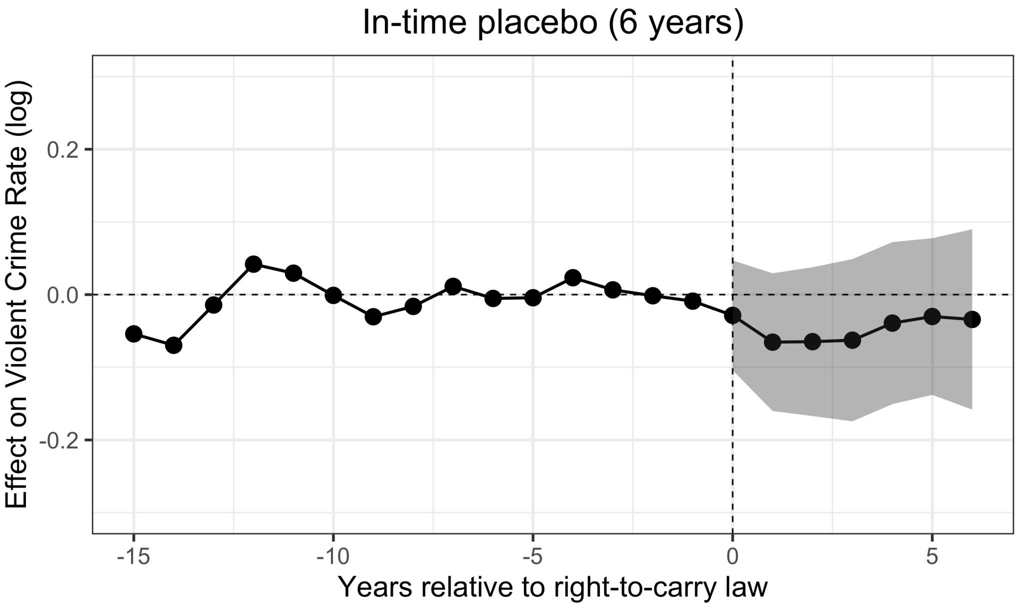


# Weights with intercept

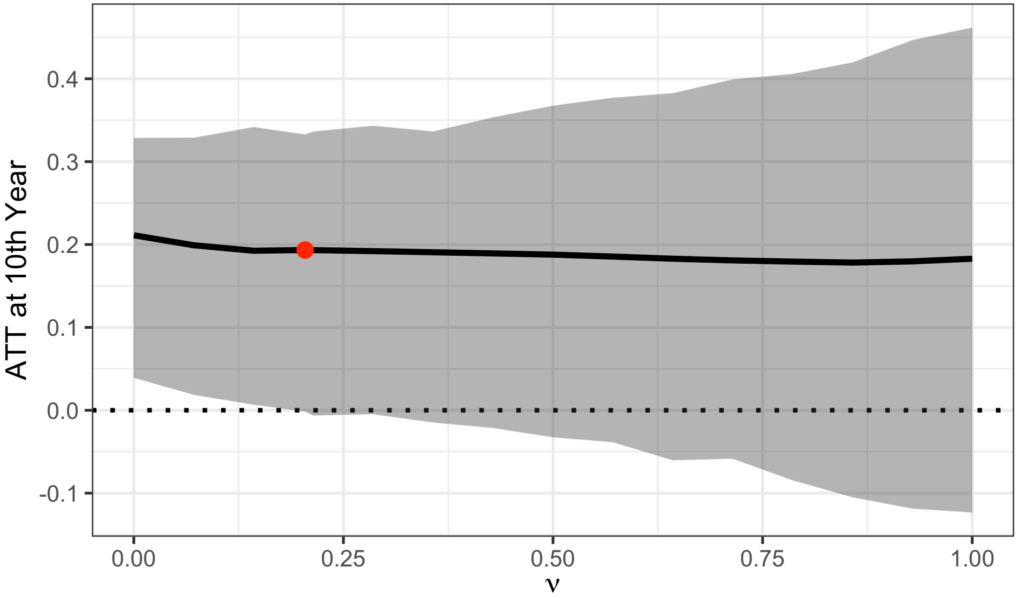



Beer consumption
Black males, 15-19
Black males, 20-39
Incarceration
Other males, 15-19
Other males, 20-39
Personal income
Police staffing
Pop in MSAs
Poverty
Unemployment
White males, 15-19
White males, 20-39
0
2
4
Absolute difference
Before weighting
W/o covariates
With covariates

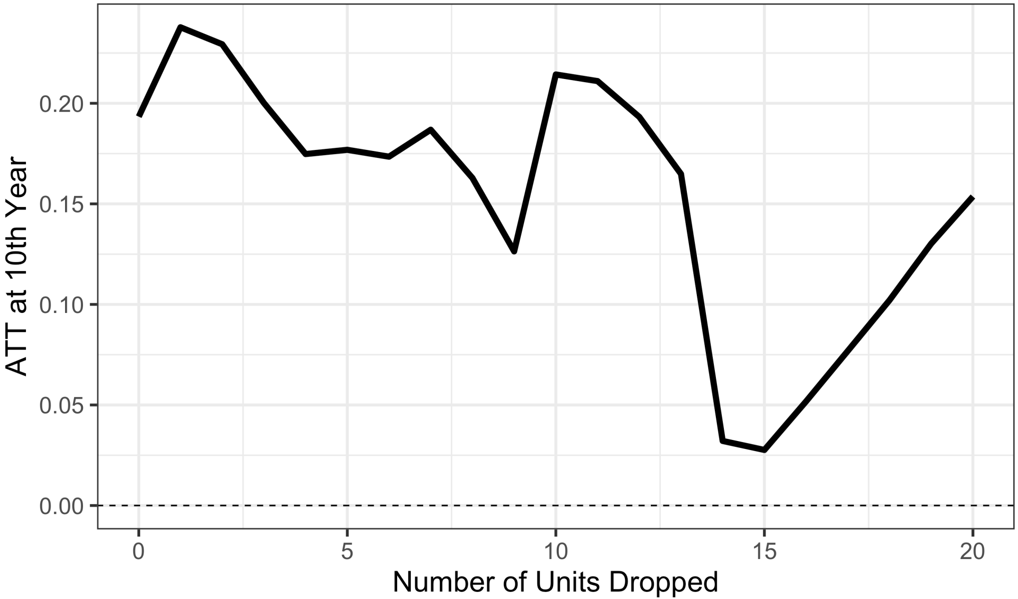

In-time placebo (2 years)
Effect on Violent Crime Rate (log)
0.2
0.0
-0.2
-15
-10
-5
0
Years relative to right-to-carry law


In-time placebo (6 years)
Effect on Violent Crime Rate (log)
0.2
0.0
-0.2
-15
-10
-5
0
5
Years relative to right-to-carry law

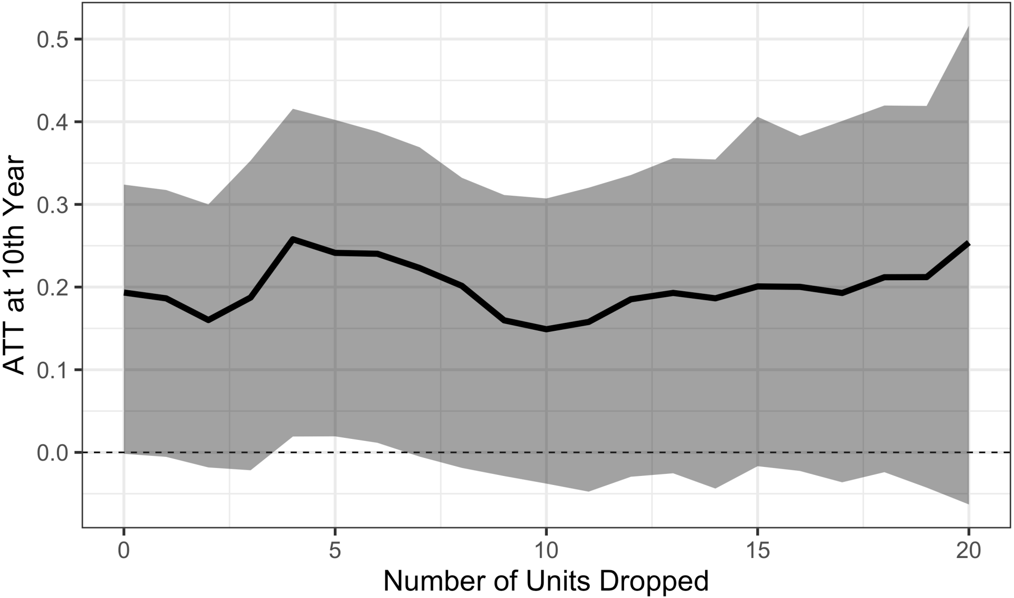
# Sensitivity to choice of $\nu$



## Dropping worst-fit units: P. Pooled SCM



# Dropping worst-fit units: P. Pooled SCM + Intercept + Covariates



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