Synthetic Controls
with Staggered Adoption

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What is the impact of right-to-carry laws on violent crime?

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What is the impact of right-to-carry laws on violent crime?

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- “More guns, less crime”?
  [Lott and Mustard, 1997]

- New research says no
  [Donohue et al., 2019]
Estimating effects under staggered adoption

*Staggered adoption:* Multiple units adopt treatment over time
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*Common approaches can fail:* Little guidance when this happens
- Difference in Differences (DiD) requires parallel trends assumption
- Synthetic Control Method (SCM) designed for single treated unit
Estimating effects under staggered adoption

*Staggered adoption:* Multiple units adopt treatment over time

*Common approaches can fail:* Little guidance when this happens
  - *Difference in Differences (DiD)* requires parallel trends assumption
  - *Synthetic Control Method (SCM)* designed for single treated unit

*Partially pooled SCM*
  - Modify optimization problem to target overall and state-specific fit
  - Account for level differences with *Intercept-Shifted SCM*
What do we want to estimate?

Units: \(i = 1, \ldots, N, J\) total treated units

Time: \(t = 1, \ldots, T,\) treatment times \(T_1, \ldots, T_J, \infty\)

Outcome: at event time \(k, Y_{i,T_j+k}\)

- Some assumptions to write down potential outcomes
  [Athey and Imbens, 2018; Imai and Kim, 2019]
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Basic building block:

\[
\tau_{jk} = Y_{jT_j+k}(T_j) - \left( \hat{\gamma}_{ij} Y_{iT_j+k} \right) \sum \hat{\gamma}_{ij} Y_{iT_j+k}
\]
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Outcome: at event time \( k \), \( Y_{i, T_j+k} \)

- Some assumptions to write down potential outcomes
  [Athey and Imbens, 2018; Imai and Kim, 2019]

Basic building block:

\[
\tau_{jk} = Y_{jT_j+k}(T_j) - \frac{\sum \hat{\gamma}_{ij} Y_{iT_j+k}}{\sum \hat{\gamma}_{ij}}
\]

Average at event time \( k \):

\[
\text{ATT}_k = \frac{1}{J} \sum_{j=1}^{J} \tau_{jk}
\]
Separate Synthetic Controls
The graph shows the log violent crime rate over years relative to the right-to-carry law (2004). The data exhibits a trend with fluctuations, indicating changes in violent crime rates before and after the law's implementation. The vertical dashed line at 0 marks the year the law was enacted.
\[
\min_{\gamma \in \Delta^{\text{scm}}} \left\| Y_{\text{OH} \ell} - \sum_{i \neq \text{OH}} \gamma_i Y_{i \ell} \right\|_2^2 + \text{penalty}
\]
\[
\min_{\gamma \in \Delta^{scm}} \left\| Y_{OH} \ell - \sum_{i \neq OH} \gamma_i Y_{i\ell} \right\|^2 + \text{penalty}
\]
\[
\min_{\gamma \in \Delta^{scm}} \| \text{State Balance} \|_2^2 + \text{penalty}
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Separate SCM

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\]
Partially Pooled SCM
\[
\min_{\Gamma \in \Delta_{scm}} \frac{1}{J} \sum_{j=1}^{J} \left\| \text{State Balance}_j \right\|_2^2 + \text{penalty}
\]
\[
\min_{\Gamma \in \Delta_{scm}} \left\| \frac{1}{J} \sum_{j=1}^{J} \text{State Balance}_j \right\|_2^2 + \text{penalty}
\]
\[
\min_{\Gamma \in \Delta_{\text{scm}}} \| \text{Pooled Balance} \|_2^2 + \text{penalty}
\]
Pooled Balance is better!

SCM pre-treatment imbalance

Pooled SCM vs. Separate SCM for different states.
Pooled Balance is better!

... but State Balance is worse
  - Bad for state estimates
Pooled Balance is better!

... but State Balance is worse
  - Bad for state estimates

Also bad for the average!
  - When DGP varies over time
Pooled Balance is better!

... but State Balance is worse
- Bad for state estimates

Also bad for the average!
- When DGP varies over time

Find weights that balance both Pooled Balance and State Balance
\[
\min_{\Gamma \in \Delta_{\text{scm}}} \nu \| \text{Pooled Balance} \|_2^2 + \frac{1 - \nu}{J} \sum_{j=1}^{J} \| \text{State Balance}_j \|_2^2 + \text{penalty}
\]
\[
\min_{\Gamma \in \Delta^{scm}} \nu \|\text{Pooled Balance}\|_2^2 + \frac{1 - \nu}{J} \sum_{j=1}^{J} \|\text{State Balance}_j\|_2^2 + \text{penalty}
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\]

Balance possibility frontier
Heuristic for $\nu = \frac{\|\text{Pooled Balance}\|_2}{\frac{1}{\sqrt{J}} \sum_{j=1}^{J} \|\text{State Balance}_j\|_2}$ fit with $\nu = 0$
Extensions
Intercept-Shifted SCM

Adjust for level differences by adding an intercept to the optimization problem

[Doudchenko and Imbens, 2017; Ferman and Pinto, 2018]

\[
\hat{Y}^*_{j,T_j+k}(\infty) = \hat{\alpha}_j + \sum_i \hat{\gamma}_{ij} Y_{i,T_j+k}
\]
Intercept-Shifted SCM

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\hat{Y}_{j,T_j+k}(\infty) = \hat{\alpha}_j + \sum_i \hat{\gamma}_{ij} Y_{i,T_j+k}
\]

Solution: De-meaning by pre-treatment average $\bar{Y}_{i,T_j}^{pre}$
Intercept-Shifted SCM

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[Doudchenko and Imbens, 2017; Ferman and Pinto, 2018]

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\hat{Y}_{j,T_j+k}^*(\infty) = \hat{\alpha}_j + \sum_i \hat{\gamma}_{ij}^* Y_{i,T_j+k}
\]

Solution: De-meaning by pre-treatment average \(\bar{Y}_{i,T_j}^{\text{pre}}\)

Treatment effect estimate is \textbf{weighted difference-in-differences}

\[
\hat{\tau}_{jk} = (Y_{j,T_j+k} - \bar{Y}_{j,T_j}^{\text{pre}}) - \sum_{i=1}^{N} \hat{\gamma}_{ij}^* (Y_{i,T_j+k} - \bar{Y}_{i,T_j}^{\text{pre}})
\]

→ Uniform weights recover “stacked” DiD [Abraham and Sun, 2018]

→ Similar in form to P-score weighted DiD [Abadie, 2005; Callaway and Sant’Anna, 2020]
Balance possibility frontier

- Separate SCM
- Partially Pooled SCM
- Pooled SCM

DiD
Incorporating auxiliary covariates

Often have additional covariates other than the main outcome
- E.g. poverty, unemployment, incarceration, and police staffing rates
- Demographics

Same trade-off between State Balance and Pooled Balance

We focus on fixed covariates, but time-varying covariates are similar
Recap

*This paper:* Extend SCM to staggered adoption
- Find weights that control *State Balance* and *Pooled Balance*
- Include an *intercept* to adjust for level differences
- Incorporate auxiliary covariates
Recap

*This paper:* Extend SCM to staggered adoption
- Find weights that control State Balance and Pooled Balance
- Include an intercept to adjust for level differences
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Thank you!
https://arxiv.org/abs/1912.03290
https://github.com/ebenmichael/augsynth
Appendix
The role of **State Balance** and **Pooled Balance**

Generalization of parallel trends: Linear Factor Model

\[ Y_{it}(\infty) = \phi_i' \mu_t + \varepsilon_{it} \]
The role of State Balance and Pooled Balance

Generalization of parallel trends: Linear Factor Model

\[ Y_{it}(\infty) = \phi_i \mu_t + \varepsilon_{it} \]

Error for ATT

\[
\left| \hat{\text{ATT}}_0 - \text{ATT}_0 \right| \lesssim \| \bar{\mu} \|_2 \| \text{Pooled Balance} \|_2 + S \sqrt{\sum_{j=1}^{J} \| \text{State Balance}_j \|_2^2 + \frac{\sqrt{\log NJ}}{T}}
\]

Level of heterogeneity over time is important

- \( \bar{\mu} \) is the average factor value \( \rightarrow \) importance of Pooled Balance
- \( S \) is the factor standard deviation \( \rightarrow \) importance of State Balance
- Special case: unit fixed effects, only Pooled Balance matters
Simulation study

![Graphs showing the Mean Absolute Deviation: Individual Estimates for different models: Two-way Fixed Effects, Factor Model, and Autoregressive Model. The graphs compare different estimation methods: DiD, Factor Model, P. Pooled SCM w/Intercept, and Partially Pooled SCM.](image)
Partially pooled SCM weights
Weights with intercept

Donor State

Treated State
Sensitivity to choice of $\nu$

The graph illustrates the sensitivity of ATT at 10th Year to the choice of $\nu$. The black line represents the estimated ATT, and the shaded area indicates the uncertainty. The value of $\nu$ affects the ATT significantly, with a higher $\nu$ leading to a lower ATT. The red dot indicates a specific value of $\nu$ that is within the uncertainty range.
Dropping worst-fit units: P. Pooled SCM + Intercept + Covariates


