Synthetic Controls and Weighted Event Studies with Staggered Adoption

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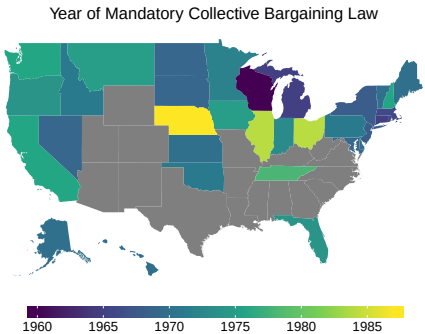
UC Berkeley

ESWC 2020

July/August 2020

What is the impact of teacher unions on salaries and spending?

- *1964 – 1987:* 33 states pass mandatory collective bargaining laws



What is the impact of teacher unions on salaries and spending?

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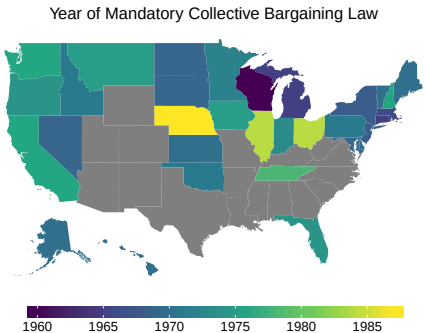
- Impact of teachers unions unclear
  - ↑ Increase expenditures by **12%** [Hoxby, 1996]
  - ↔ Or really no effect at all? [Paglayan, 2019]



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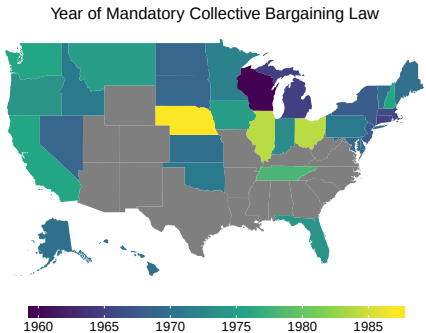
- What should we believe?



Estimating effects under staggered adoption

Staggered adoption: Multiple units adopt treatment over time

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- Difference in Differences (DiD) requires parallel trends assumption
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Partially pooled SCM

- Modify optimization problem to target overall and state-specific fit

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Weighted Event Study

- Combine outcome modeling/DiD and SCM

What do we want to estimate?

Units: $i = 1, \ldots, N$, J total treated units

Time: $t = 1, \ldots, T$, treatment times $T_1, \ldots, T_J, \infty$

Outcome: at *event time* k, Y_{i,T_j+k}

- Some assumptions to write down potential outcomes
  [Athey and Imbens, 2018; Imai and Kim, 2019]

$$\text{treat} = \begin{pmatrix} & \checkmark & \checkmark & \checkmark \\ & & \checkmark & \checkmark \\ & & & \checkmark \\ & & & \end{pmatrix}$$

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Basic building block:

$$\tau_{jk} = Y_{j,T_j+k}(1) \quad - \quad \underbrace{Y_{j,T_j+k}(0)}_{\sum \hat{\gamma}_{ij} Y_{i,T_j+k}}$$

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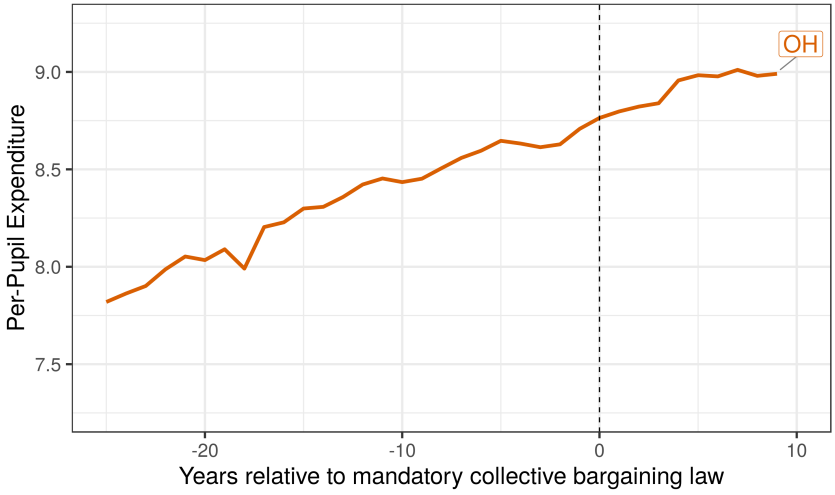
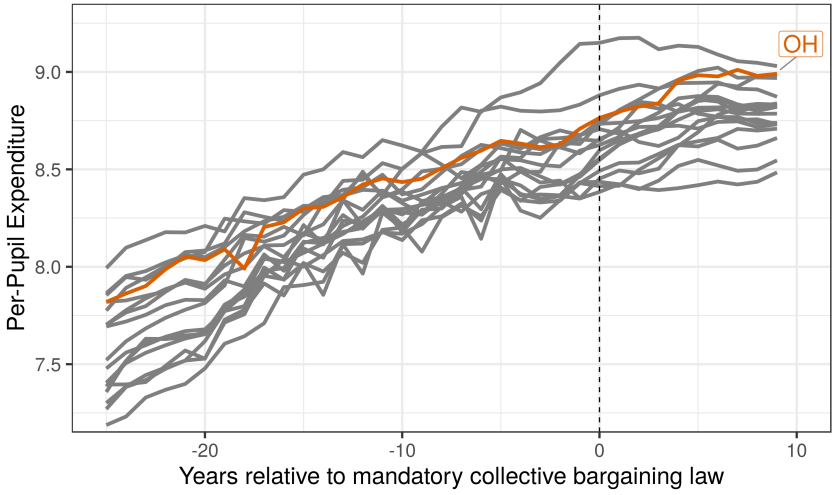
Average at event time k:

$$\text{ATT}_k = \frac{1}{J} \sum_{j=1}^{J} \tau_{jk}$$

Separate SCM


OH
9.0
8.5
8.0
7.5
Per-Pupil Expenditure
-20
-10
0
10
Years relative to mandatory collective bargaining law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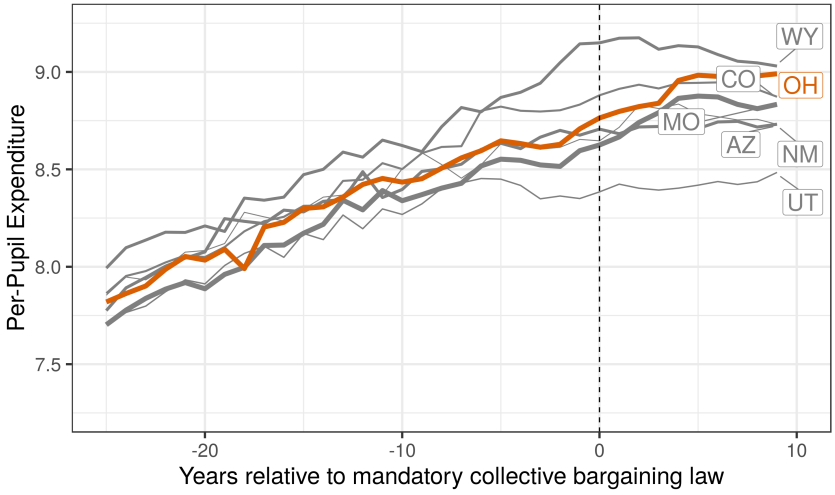
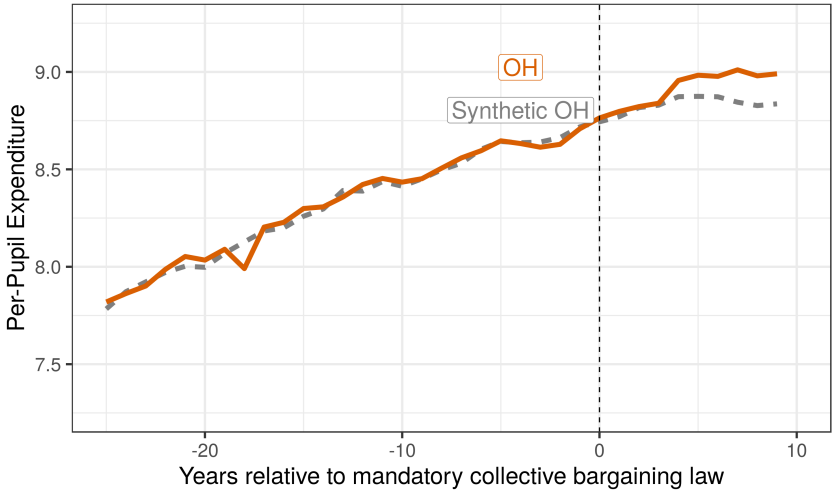


$$\min_{\gamma_j \in \Delta_j^{\text{scm}}} \|\text{State Balance}_j\|_2^2$$



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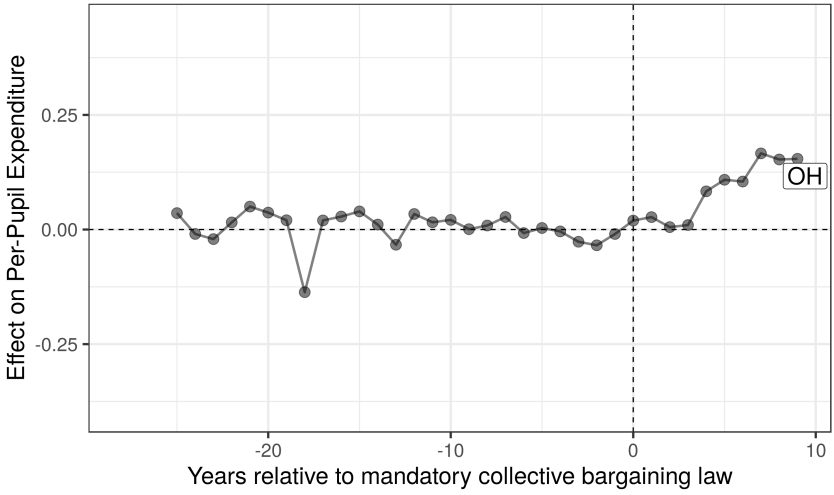

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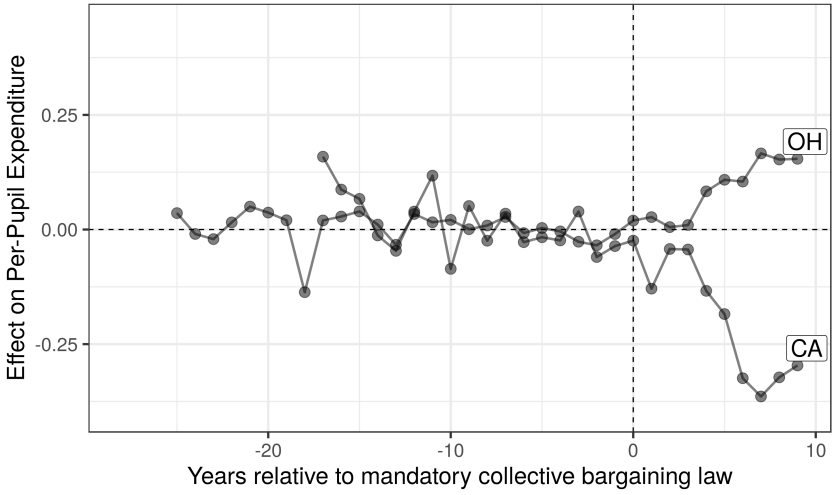

$$\min_{\gamma_1,\ldots,\gamma_J \in \Delta^{\text{scm}}} \frac{1}{J} \sum_{j=1}^{J} \|\text{State Balance}_j\|_2^2$$

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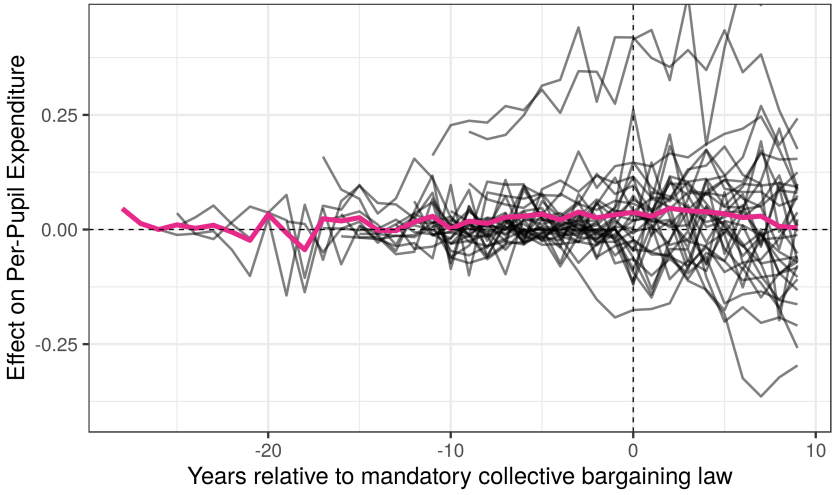

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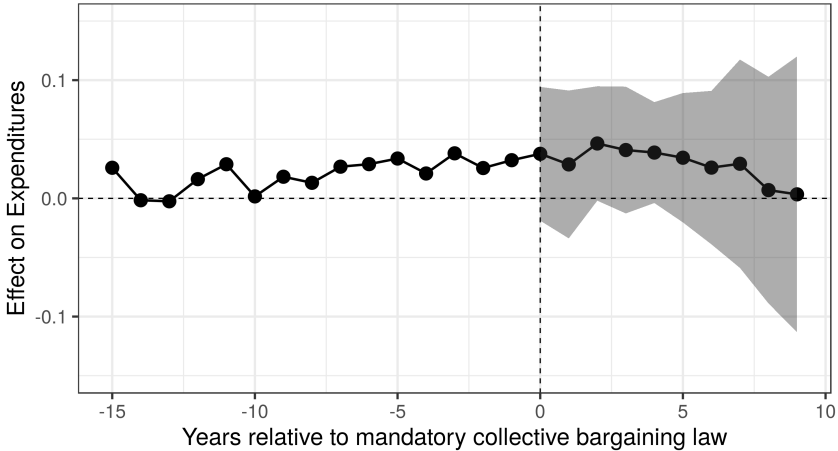
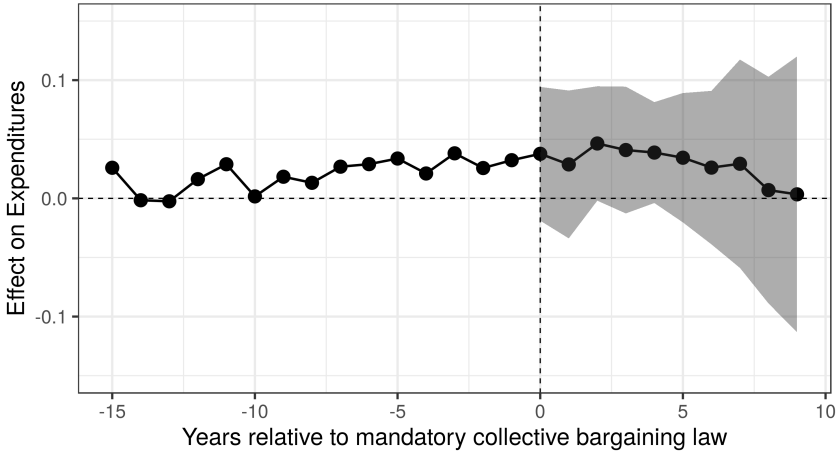
Separate SCM



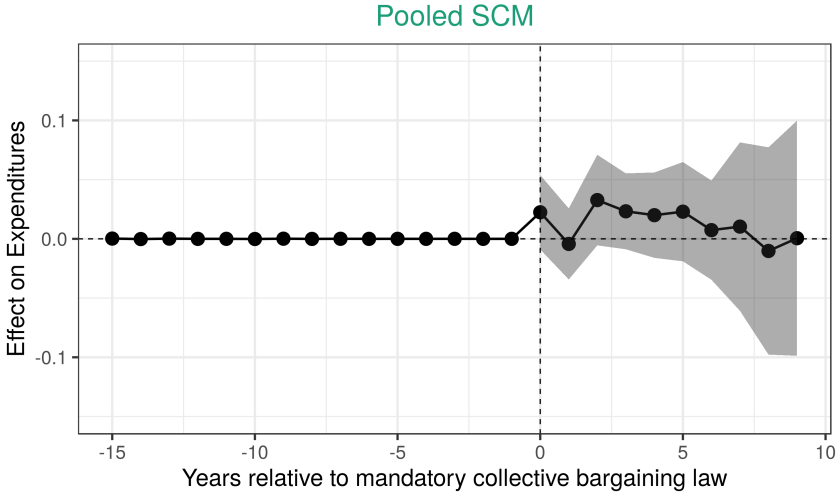
$$\min_{\gamma_1,\ldots,\gamma_J \in \Delta^{\text{scm}}} \frac{1}{J} \sum_{j=1}^{J} \|\text{State Balance}_j\|_2^2$$

Moving Beyond Separate SCM

Separate SCM

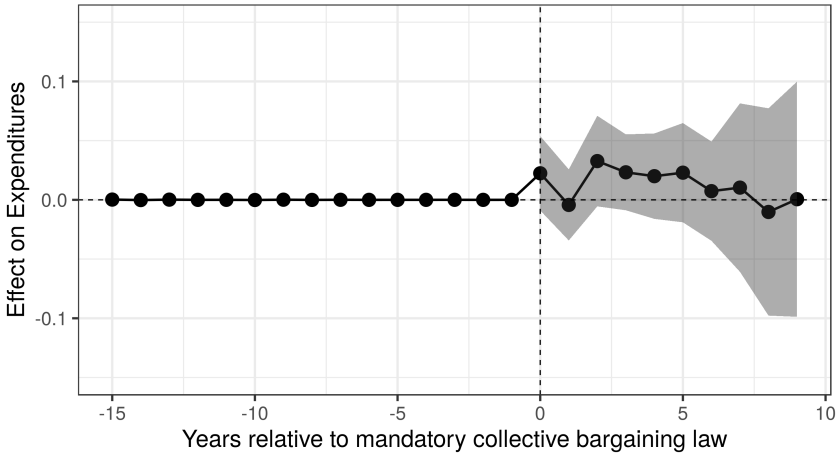


$$\min_{\Gamma} \frac{1}{J} \sum_{j=1}^{J} \|\text{State Balance}_j\|_2^2$$

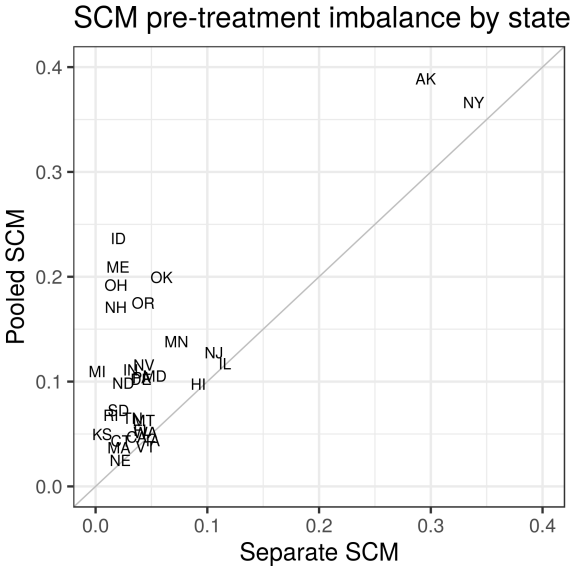


$$\min_{\Gamma} \left\| \frac{1}{J} \sum_{j=1}^{J} \text{State Balance}_j \right\|_2^2$$

Pooled SCM



$$\min_{\Gamma} \|\text{Avg Balance}\|_2^2$$



- Avg Balance is better
- but State Balance is worse.

Which matters more?

Generalization of parallel trends: Linear Factor Model

$$Y_{it}(0) = \phi_i' \mu_t + \varepsilon_{it}$$

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Generalization of parallel trends: Linear Factor Model

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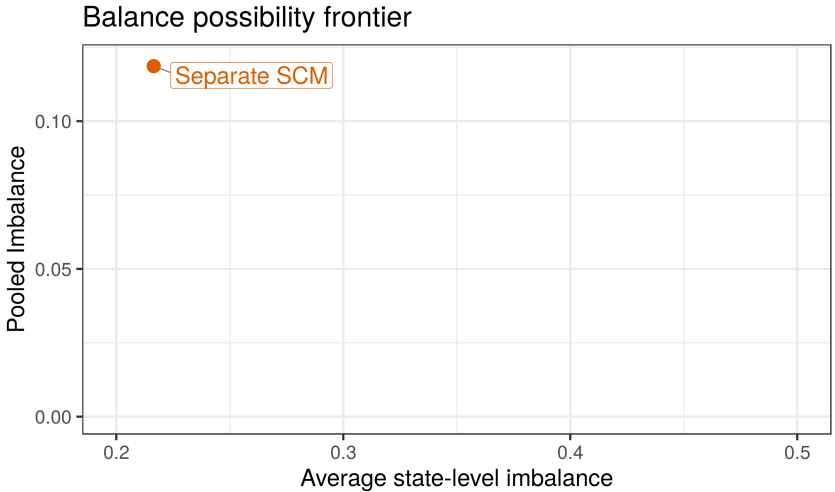
Error for ATT

$$\left| \widehat{\text{ATT}}_0 - \text{ATT}_0 \right| \lesssim \|\bar{\mu}\|_2 \|\text{Avg Balance}\|_2 + S \sqrt{\sum_{j=1}^{J} \|\text{State Balance}_j\|_2^2} + \sqrt{\frac{\log NJ}{T}}$$

Level of **heterogeneity over time** is important

- $\bar{\mu}$ is the average factor value $\rightarrow$ importance of Avg Balance
- S is the factor standard deviation $\rightarrow$ importance of State Balance
- Special case: unit fixed effects, only Avg Balance matters



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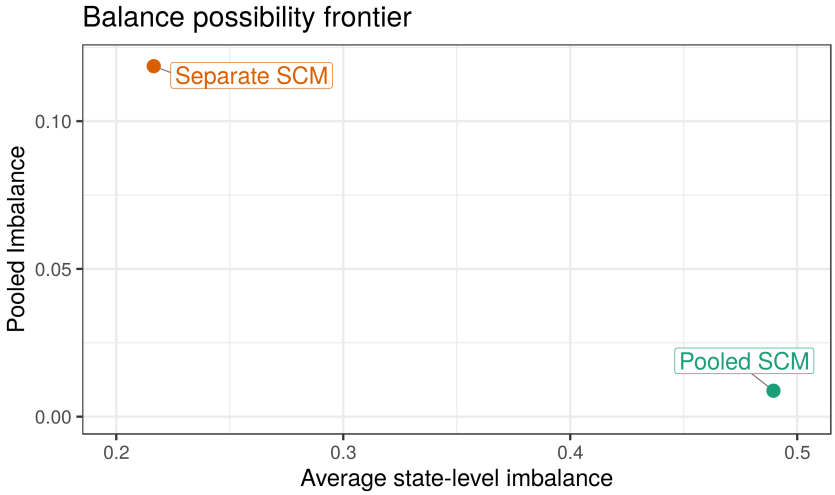

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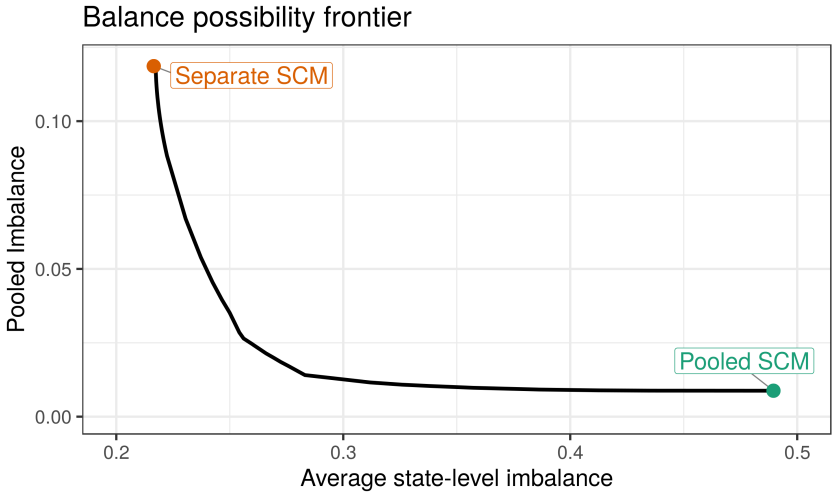
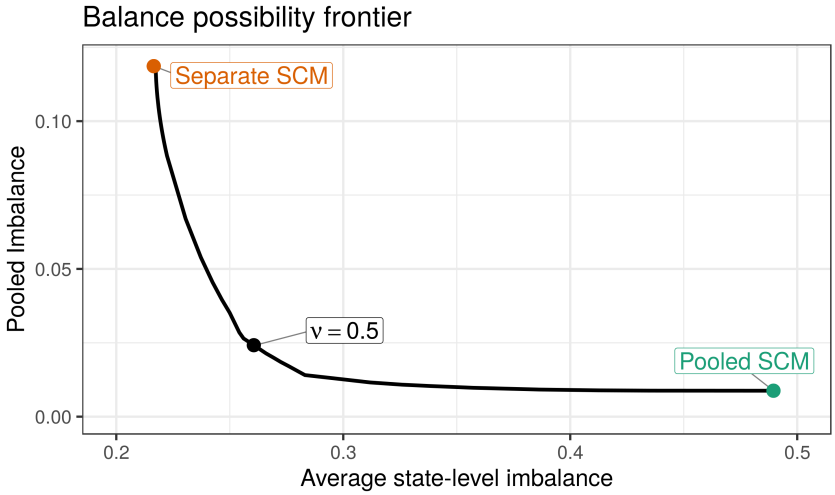


$$\min_{\Gamma} \quad \nu \, \|\text{Avg Balance}\|_2^2 + \frac{1-\nu}{J} \sum_{j=1}^{J} \|\text{State Balance}_j\|_2^2$$



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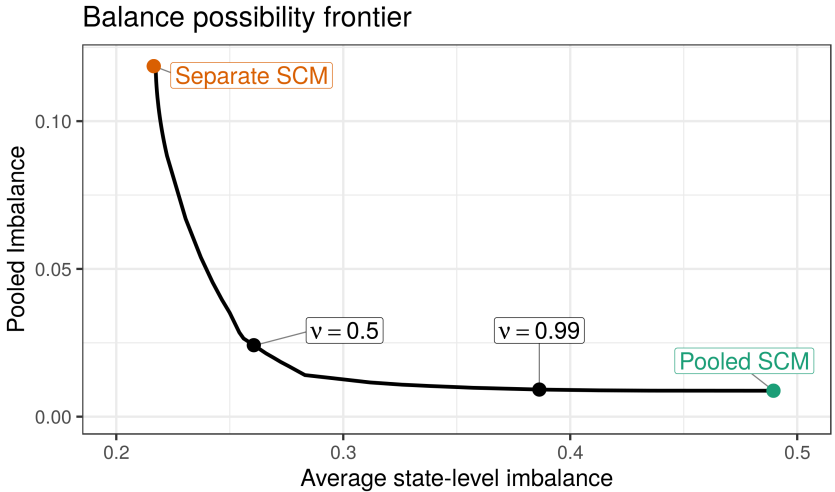

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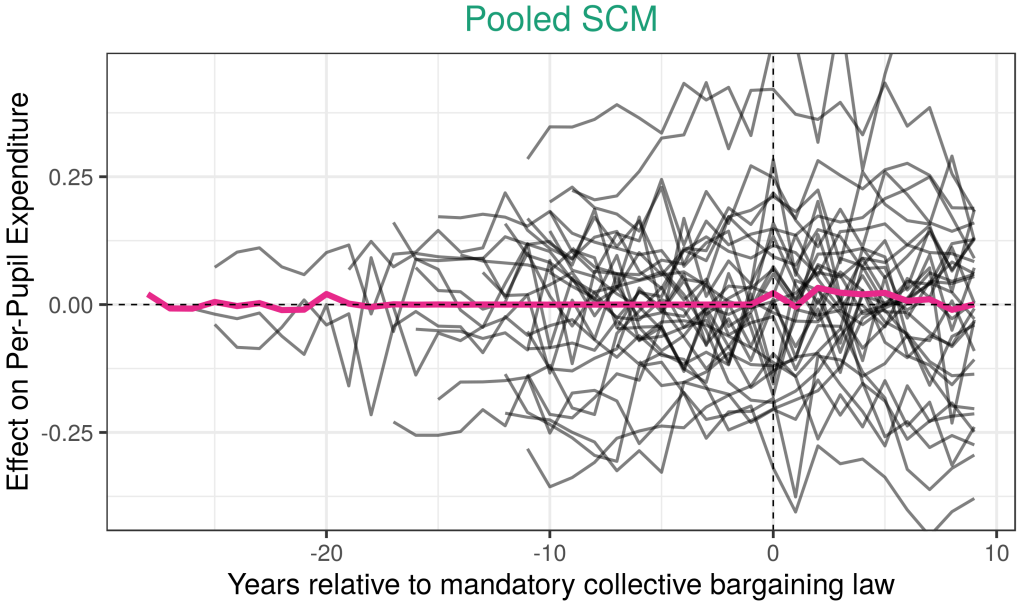
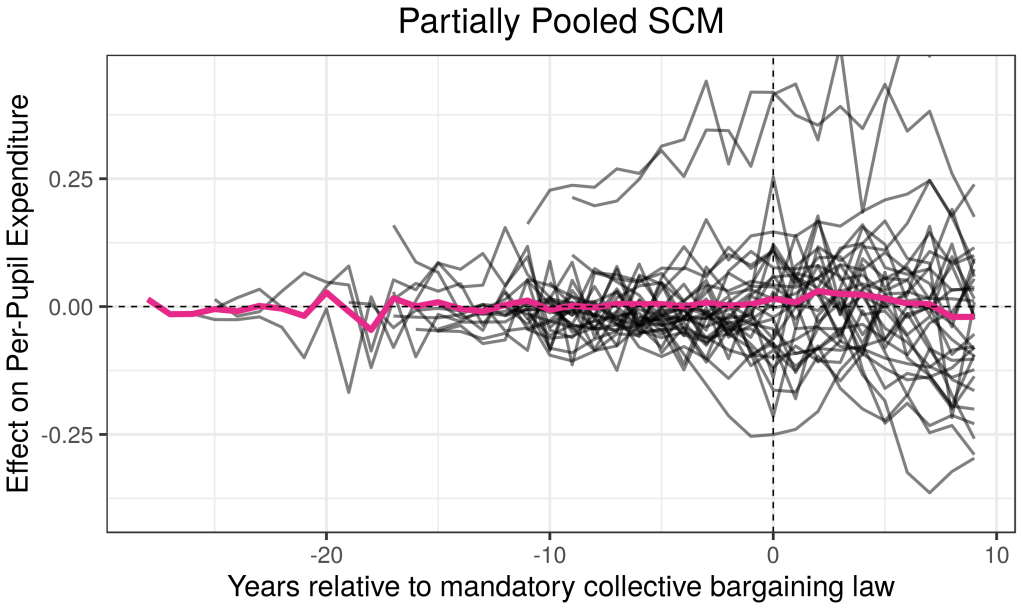

Pooled SCM
Effect on Per-Pupil Expenditure
0.25
0.00
-0.25
-20
-10
0
10
Years relative to mandatory collective bargaining law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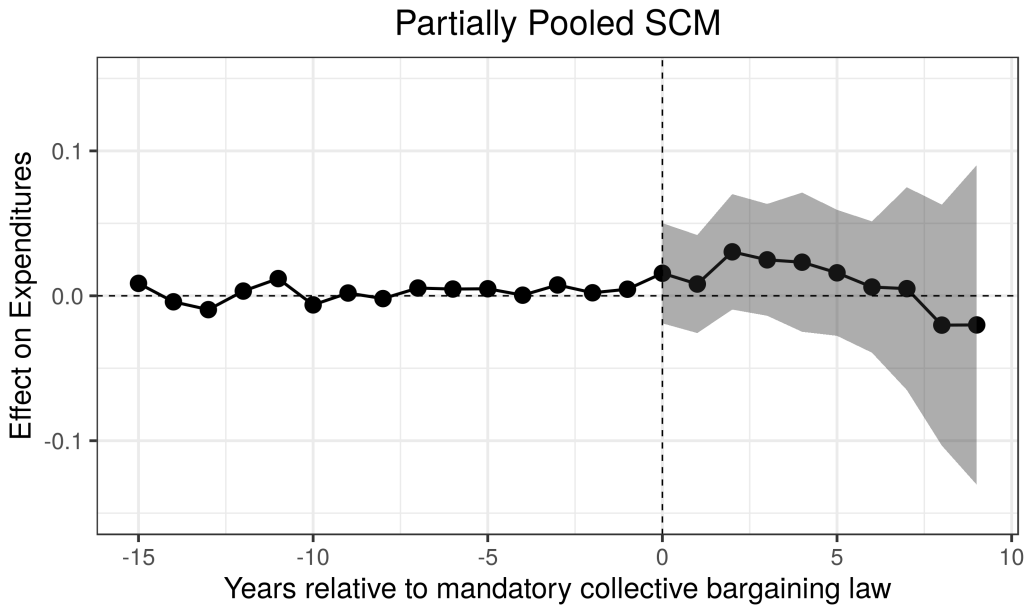
Partially Pooled SCM
Effect on Expenditures
0.1
0.0
-0.1
-15
-10
-5
0
5
10
Years relative to mandatory collective bargaining law

Combining SCM and Outcome Modeling

Weighted Event Study: Fixed Effects + SCM

- Estimate unit fixed effects via pre-treatment average $\bar{Y}^{\text{pre}}_{i,T_j}$

$$\hat{Y}^{\text{aug}}_{j,T_j+k}(0) = \bar{Y}^{\text{pre}}_{j,T_j} + \sum_{i=1}^{N} \hat{\gamma}^{*}_{ij} \left(Y_{i,T_j+k} - \bar{Y}^{\text{pre}}_{i,T_j} \right)$$

- Estimate SCM weights $\hat{\gamma}^{*}_{ij}$ using **residuals**, equivalent to adding an intercept
  [Doudchenko and Imbens, 2017; Ferman and Pinto, 2018]

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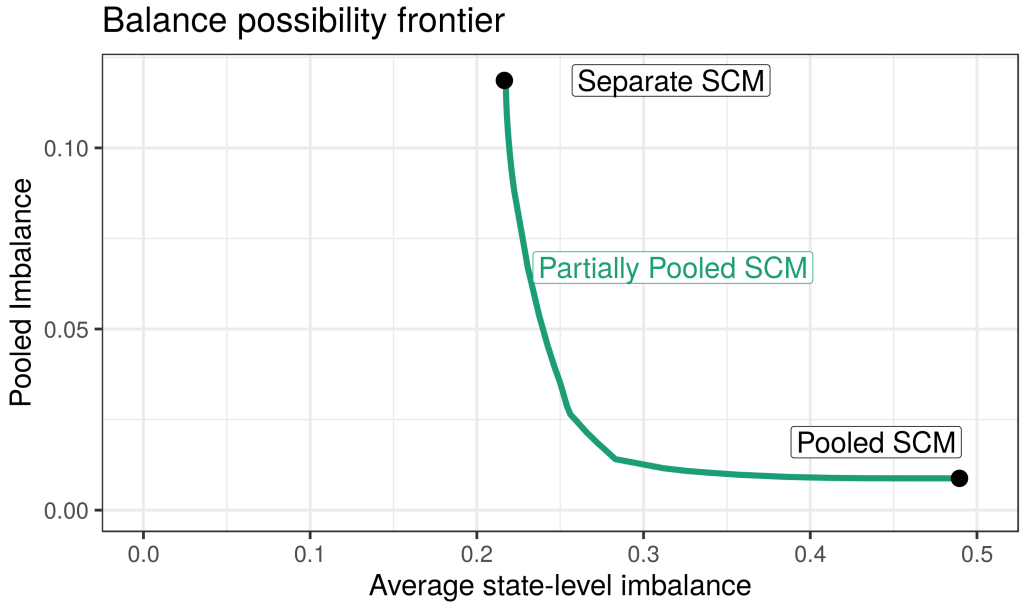
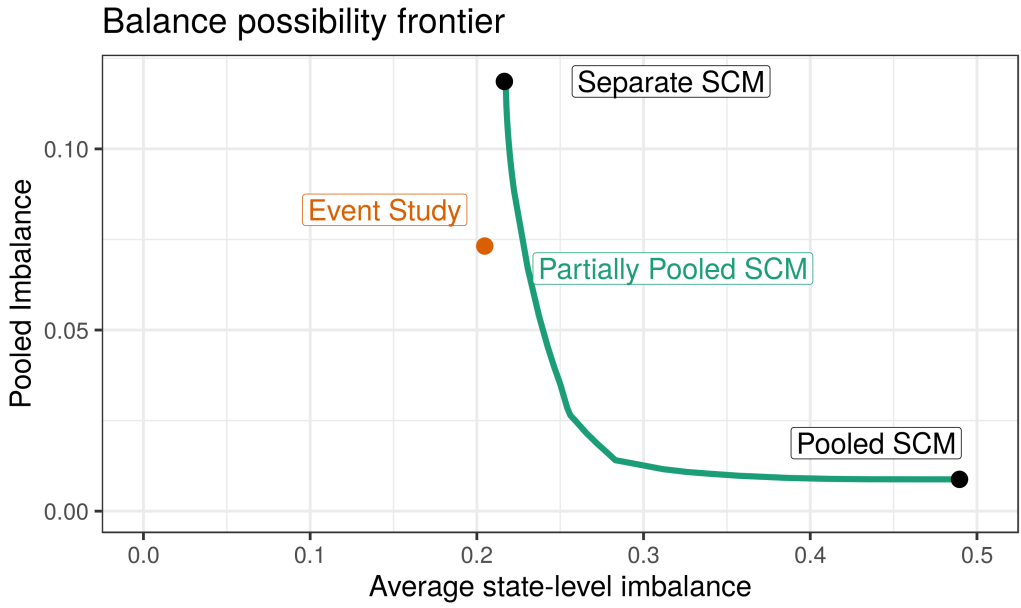
- Treatment effect estimate is **weighted difference-in-differences**

$$\hat{\tau}^{\text{aug}}_{jk} = \left(Y_{j,T_j+k} - \bar{Y}^{\text{pre}}_{j,T_j} \right) - \sum_{i=1}^{N} \hat{\gamma}^{*}_{ij} \left(Y_{i,T_j+k} - \bar{Y}^{\text{pre}}_{i,T_j} \right)$$

  → Uniform weights recover "stacked" DiD [Abraham and Sun, 2018; Callaway and Sant'Anna, 2018]
  → Extends to generic panel models [Ben-Michael et al., 2019]


Balance possibility frontier
Separate SCM
Partially Pooled SCM
Pooled SCM
Pooled Imbalance
0.10
0.05
0.00
0.0
0.1
0.2
0.3
0.4
0.5
Average state-level imbalance


Balance possibility frontier
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Event Study
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0.10
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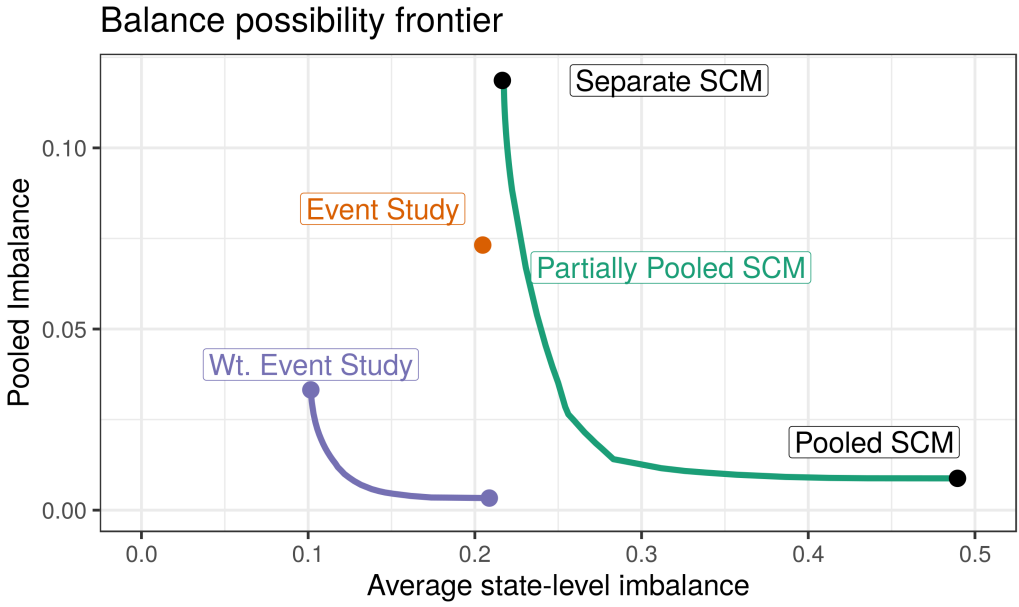
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Partially Pooled SCM
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Average state-level imbalance

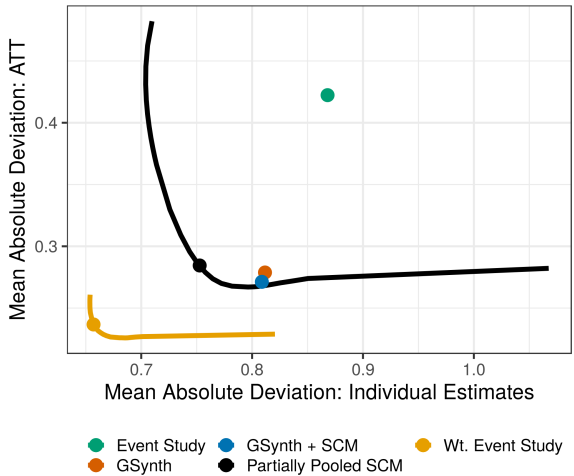
Factor model simulation

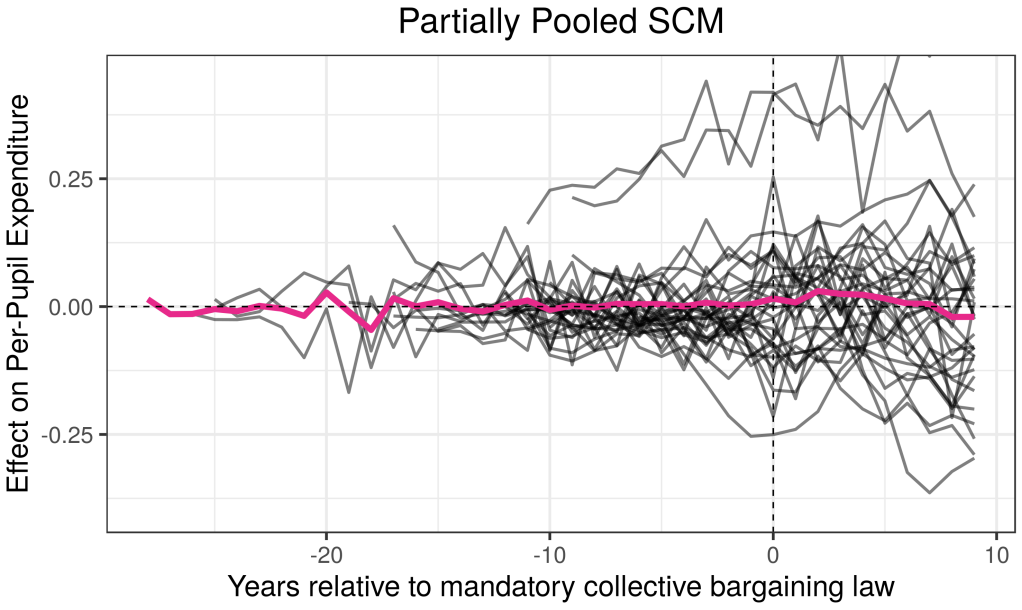
Calibrated sim study:

- Fit `gsynth` [Xu, 2017]

$$Y_{it} = \text{unit}_i + \text{time}_t + \phi_i' \mu_t + \varepsilon_{it}$$

- $\{\text{unit}_i, \phi_i\} \sim \widehat{\text{MVN}}$
- $\pi_i = \text{logit}(\theta_0 + \theta_1(\text{unit}_i + \phi_{i1} + \phi_{i2}))$

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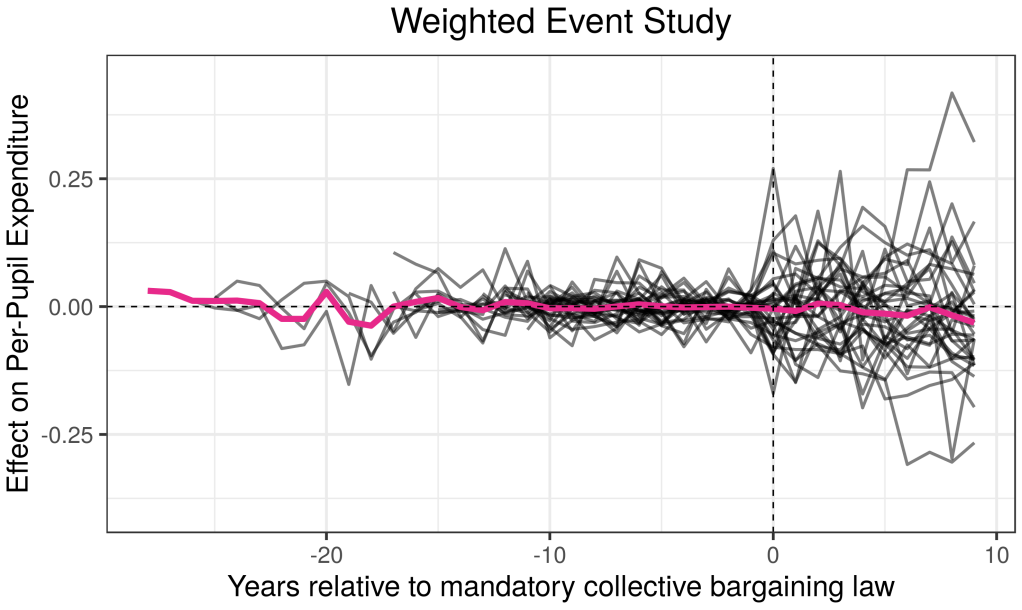
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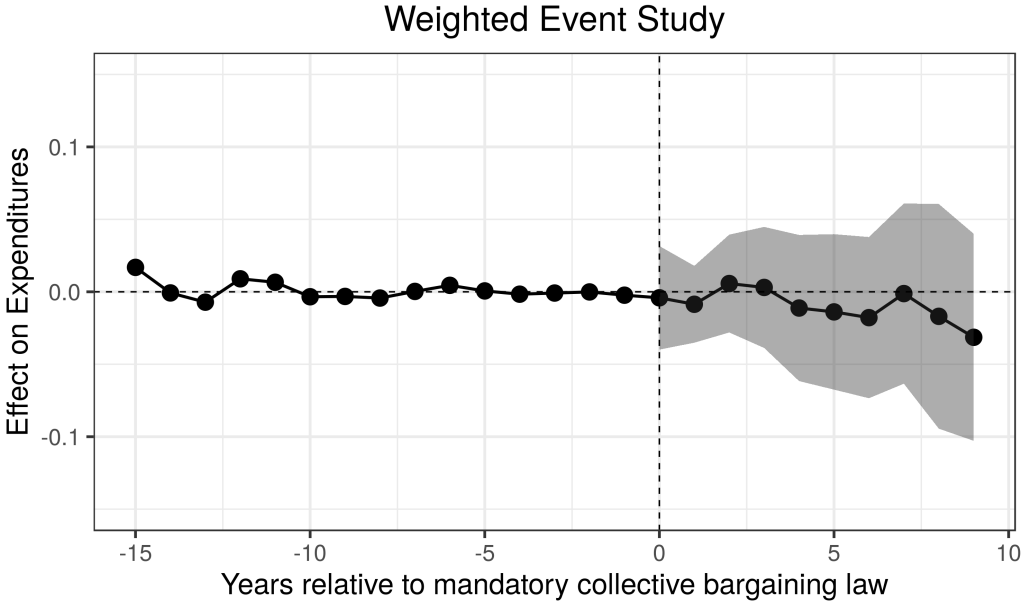
Weighted Event Study
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Recap

This paper: Extend SCM to staggered adoption

- Find weights that control State Balance and Avg Balance
- Combine SCM and outcome modeling to improve over both
- *Under the hood:* Dual shrinkage; connection to (generalized) IPW

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- Combine with regression, general outcome models
- Extend to unbalanced panels
- Sensitivity analysis

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Thank you!

https://arxiv.org/abs/1912.03290

https://github.com/ebenmichael/augsynth

Appendix

Random effects AR simulation: level of pooling matters more

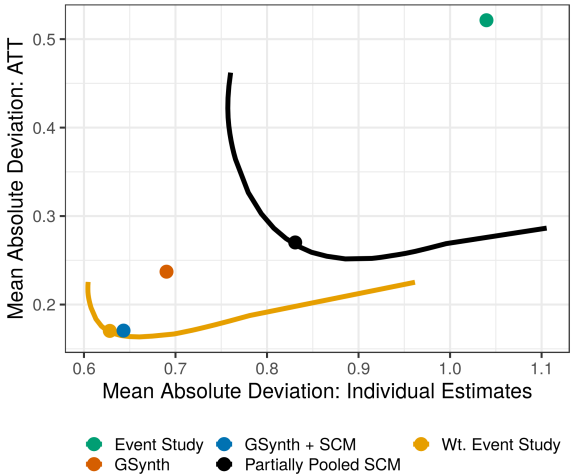
Calibrated sim study: Random Effects AR

- Fit random effects model
  [Gelman and Hill, 2007]

$$Y_{it} = \sum_{k=1}^{3} \rho_{tk} Y_{i(t-k)} + \varepsilon_{it}$$

$$\rho_t \sim N(\bar{\rho}, \Sigma)$$

- $\pi_i = \text{logit}\left(\theta_0 + \theta_1 \sum_{k=-3}^{1} Y_{i(t-k)}\right)$



DGP is FE Model: Weighted event study performs well

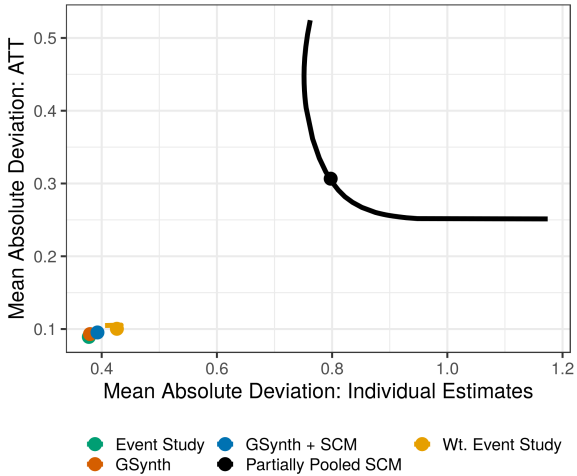
Calibrated sim study: FE

- Fit FE model

$$Y_{it} = \text{unit}_i + \text{time}_t + \varepsilon_{it}$$

- $\text{unit}_i \sim \widehat{\text{Normal}}$
- $\pi_i = \text{logit}(\theta_0 + \theta_1 \cdot \text{unit}_i)$

Event study is correct model



Heuristic for ν: fit with $\nu = 0$ then choose

$$\hat{\nu} = \frac{\frac{1}{\sqrt{L}} \left\| \text{Avg Balance} \right\|_2}{\sqrt{\frac{1}{J} \sum_{j=1}^{J} \left\| \text{State Balance}_j \right\|_2^2}}$$

References I

Abraham, S. and Sun, L. (2018). Estimating dynamic treatment effects in event studies with heterogeneous treatment effects.

Athey, S. and Imbens, G. W. (2018). Design-based analysis in difference-in-differences settings with staggered adoption. Technical report, National Bureau of Economic Research.

Ben-Michael, E., Feller, A., and Rothstein, J. (2019). The Augmented Synthetic Control Method.

Callaway, B. and Sant'Anna, P. H. C. (2018). Difference-in-Differences With Multiple Time Periods and an Application on the Minimum Wage and Employment.

Doudchenko, N. and Imbens, G. W. (2017). Difference-In-Differences and Synthetic Control Methods: A Synthesis. *arxiv 1610.07748*.

Ferman, B. and Pinto, C. (2018). Synthetic controls with imperfect pre-treatment fit.

Gelman, A. and Hill, J. (2007). *Data Analysis Using Regression and Multilevel/Hierachical Models*.

Hoxby, C. M. (1996). How teachers' unions affect education production. *The Quarterly Journal of Economics*, 111(3):671–718.

References II

Imai, K. and Kim, I. S. (2019). On the use of two-way fixed effects regression models for causal inference with panel data.

Paglayan, A. S. (2019). Public-sector unions and the size of government. *American Journal of Political Science*, 63(1):21–36.

Xu, Y. (2017). Generalized Synthetic Control Method: Causal Inference with Interactive Fixed Effects Models. *Political Analysis*, 25:57–76.