Synthetic Controls with Staggered Adoption

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UC Berkeley

Online Causal Inference Seminar

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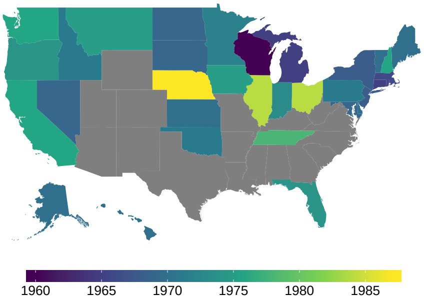
What is the impact of teacher unionization on education spending?

- *1960 – 1987:* 34 states pass mandatory collective bargaining laws

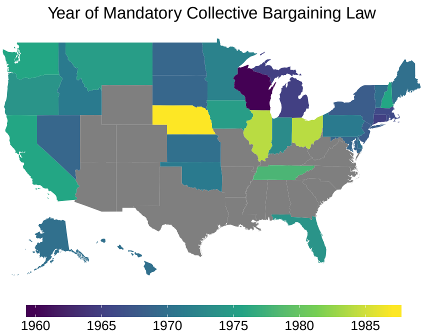
Year of Mandatory Collective Bargaining Law



What is the impact of teacher unionization on education spending?

- *1960 – 1987:* 34 states pass mandatory collective bargaining laws

- Impact of teachers unions unclear
  - ↑ Increase expenditures by **12%** [Hoxby, 1996]
  - ↔ Or really no effect at all? [Paglayan, 2019]



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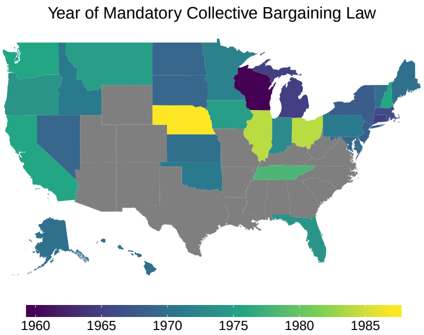
- What should we believe?



Estimating effects under staggered adoption

Staggered adoption: Multiple units adopt treatment over time

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Common approaches can fail: Little guidance when this happens

- Difference in Differences (DiD) requires parallel trends assumption
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Partially pooled SCM

- Modify optimization problem to target overall and state-specific fit
- Account for level differences with Intercept-Shifted SCM

What do we want to estimate?

Units: $i = 1, \ldots, N$, J total treated units

Time: $t = 1, \ldots, T$, treatment times $T_1, \ldots, T_J, \infty$

Outcome: at *event time* k, Y_{i,T_j+k}

– Some assumptions to write down potential outcomes
  [Athey and Imbens, 2018; Imai and Kim, 2019]

$$\text{treat} = \begin{pmatrix} & \checkmark & \checkmark & \checkmark \\ & & \checkmark & \checkmark \\ & & & \checkmark \\ & & & \end{pmatrix}$$

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Basic building block:

$$\tau_{jk} = Y_{j,T_j+k}(T_j) \quad - \quad \underbrace{Y_{j,T_j+k}(\infty)}_{\sum \hat{\gamma}_{ij} Y_{i,T_j+k}}$$

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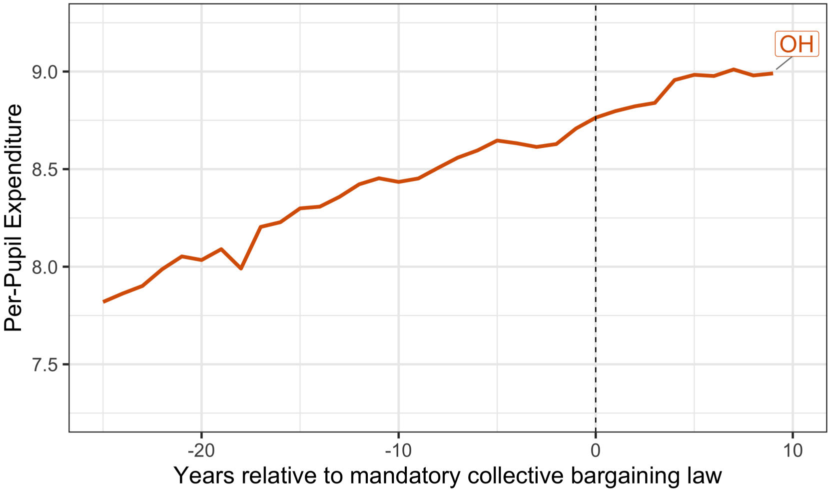
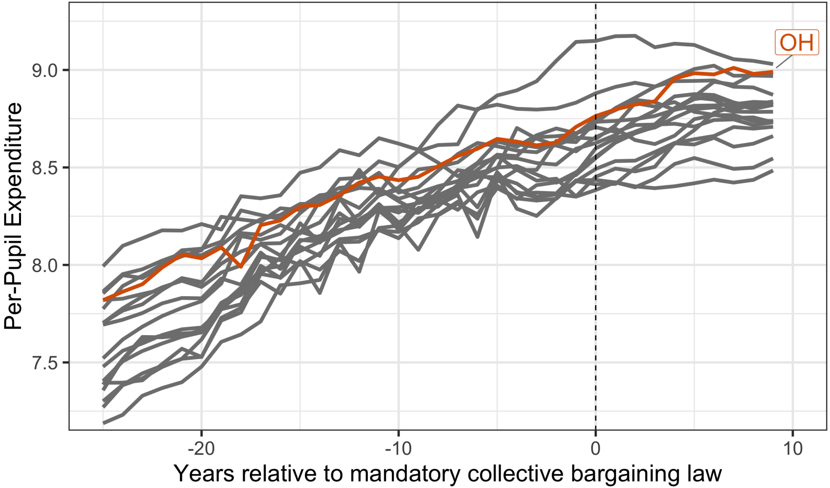
Average at event time k:

$$\text{ATT}_k = \frac{1}{J} \sum_{j=1}^{J} \tau_{jk}$$

Separate SCM


OH
9.0
8.5
8.0
7.5
Per-Pupil Expenditure
-20
-10
0
10
Years relative to mandatory collective bargaining law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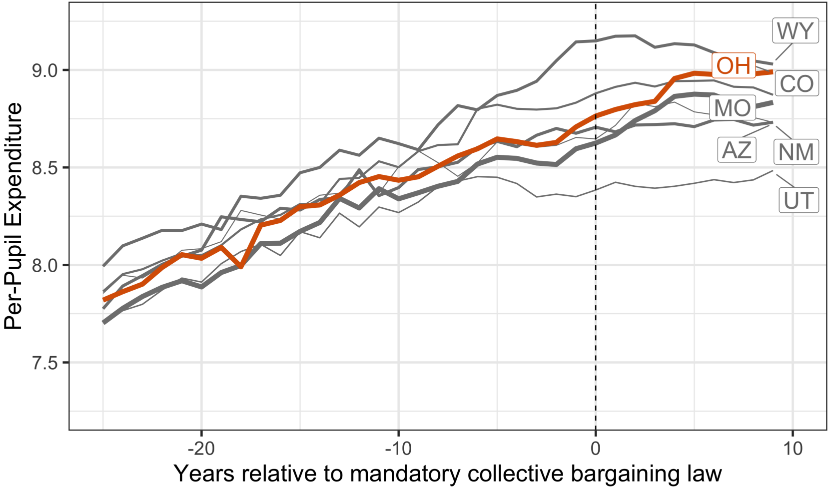
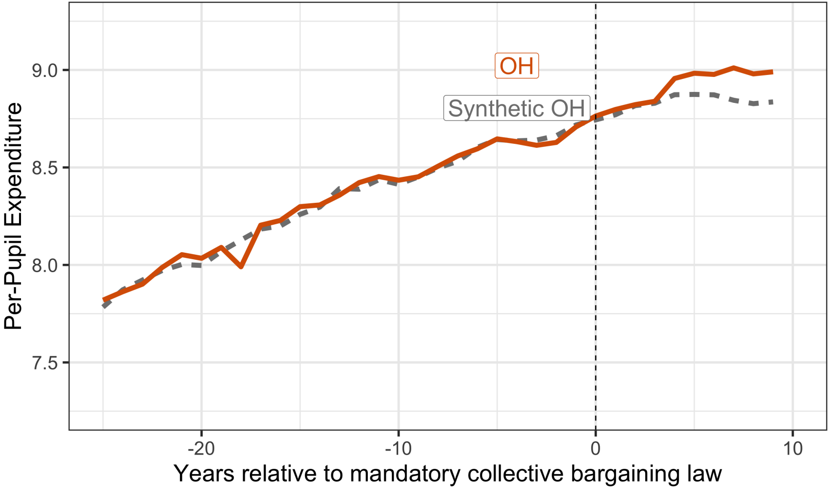


$$\min_{\gamma_j \in \Delta_j^{\text{scm}}} \quad \|\text{State Balance}_j\|_2^2$$



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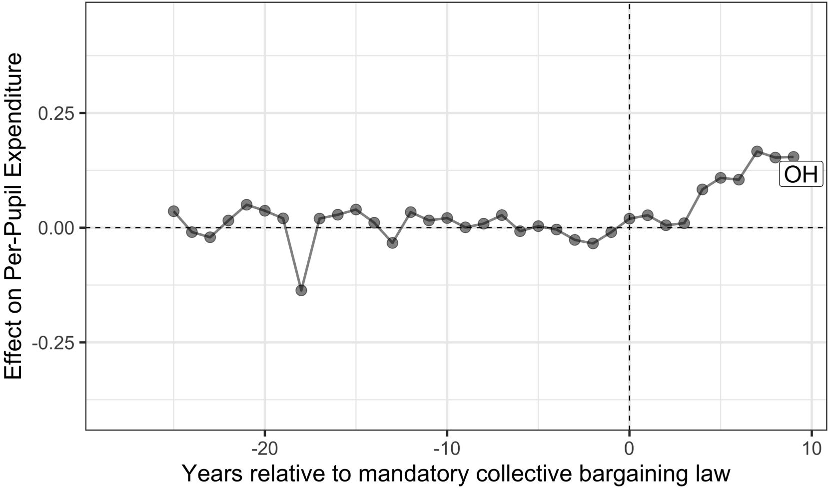

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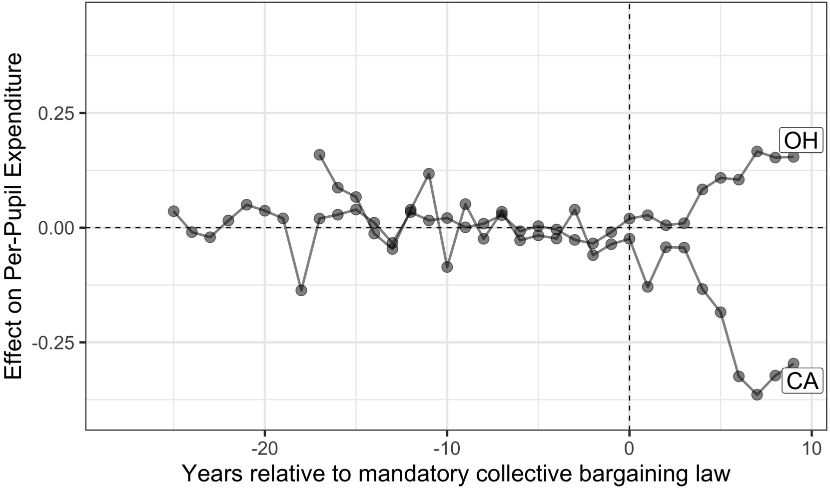
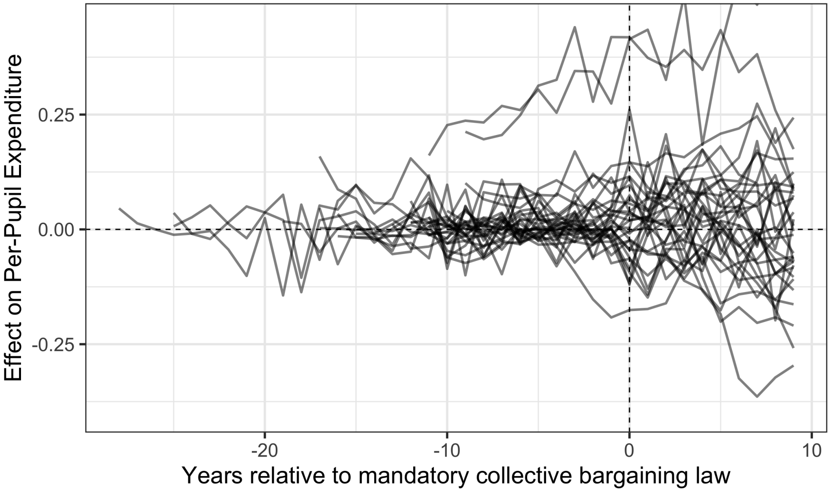


$$\min_{\gamma_1,\ldots,\gamma_J \in \Delta^{\text{scm}}} \frac{1}{J} \sum_{j=1}^{J} \|\text{State Balance}_j\|_2^2$$



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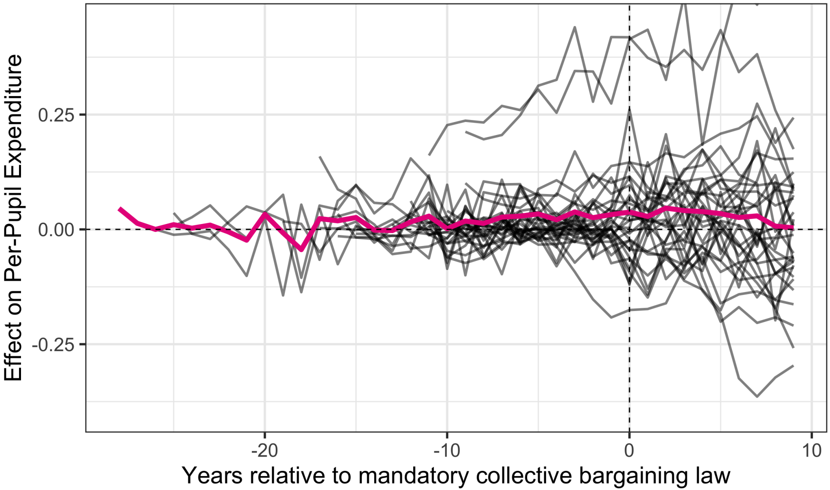

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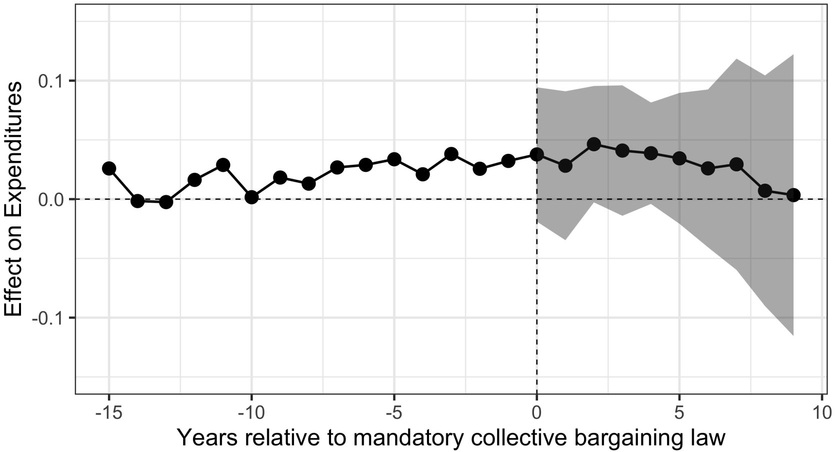
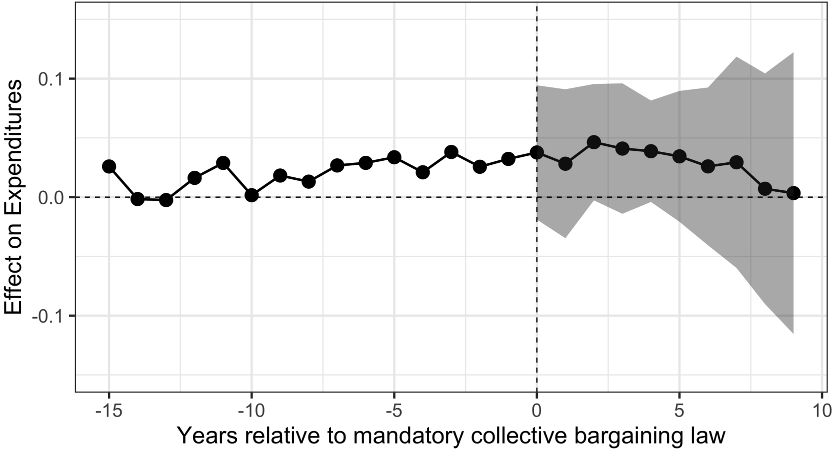


$$\min_{\gamma_1,\ldots,\gamma_J \in \Delta^{\text{scm}}} \frac{1}{J} \sum_{j=1}^{J} \|\text{State Balance}_j\|_2^2$$

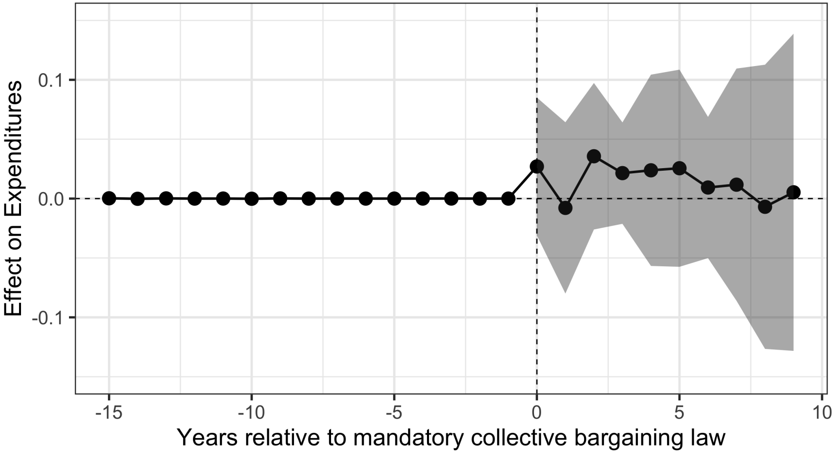
Moving Beyond Separate SCM

Separate SCM



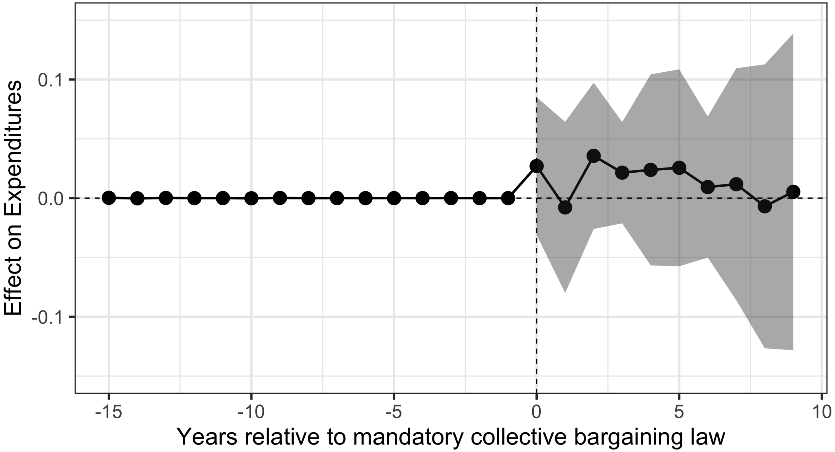
$$\min_{\Gamma} \frac{1}{J} \sum_{j=1}^{J} \|\text{State Balance}_j\|_2^2$$

Pooled SCM

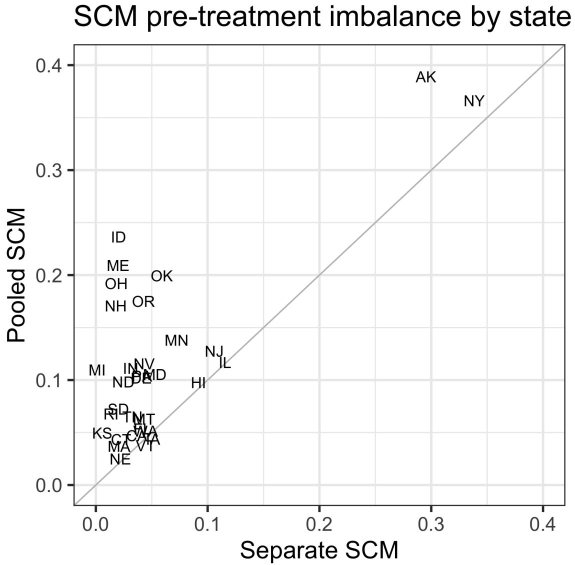


$$\min_{\Gamma} \left\| \frac{1}{J} \sum_{j=1}^{J} \text{State Balance}_j \right\|_2^2$$

Pooled SCM



$$\min_{\Gamma} \|\text{Avg Balance}\|_2^2$$



- Avg Balance is better
- but State Balance is worse.

Which matters more?

Generalization of parallel trends: Linear Factor Model

$$Y_{it}(\infty) = \phi_i' \mu_t + \varepsilon_{it}$$

Which matters more?

Generalization of parallel trends: Linear Factor Model

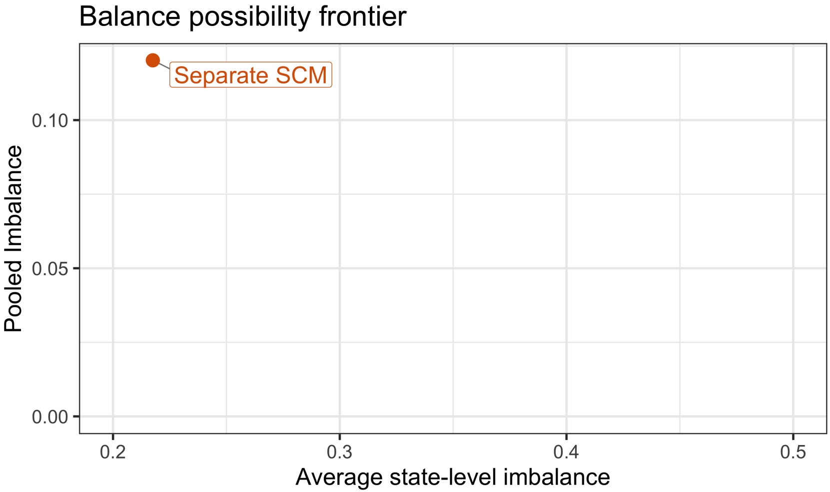
$$Y_{it}(\infty) = \phi_i' \mu_t + \varepsilon_{it}$$

Error for ATT

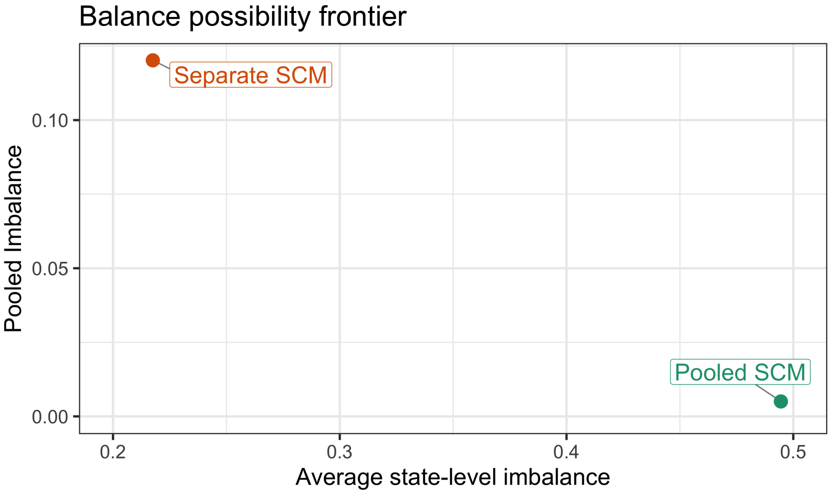
$$\left| \widehat{\text{ATT}}_0 - \text{ATT}_0 \right| \lesssim \|\bar{\mu}\|_2 \|\text{Avg Balance}\|_2 + S \sqrt{\sum_{j=1}^{J} \|\text{State Balance}_j\|_2^2} + \sqrt{\frac{\log NJ}{T}}$$

Level of **heterogeneity over time** is important

- $\bar{\mu}$ is the average factor value → importance of Avg Balance
- S is the factor standard deviation → importance of State Balance
- Special case: unit fixed effects, only Avg Balance matters



$$\min_{\Gamma} \frac{1}{J} \sum_{j=1}^{J} \|\text{State Balance}_j\|_2^2$$



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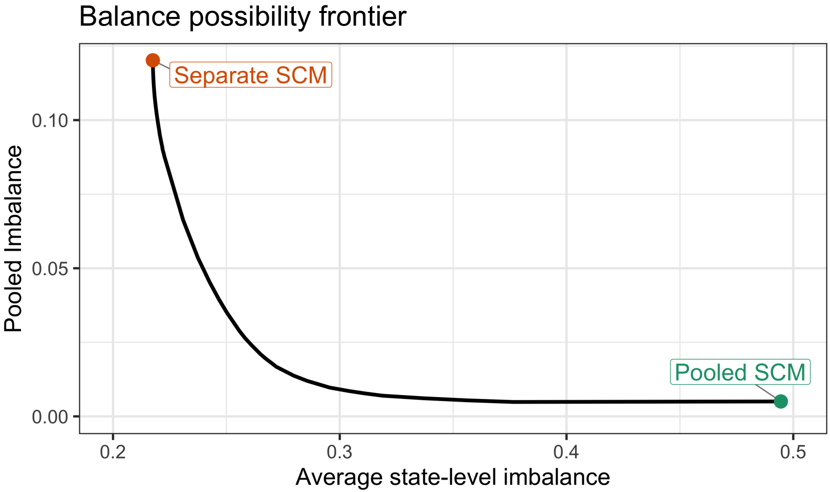
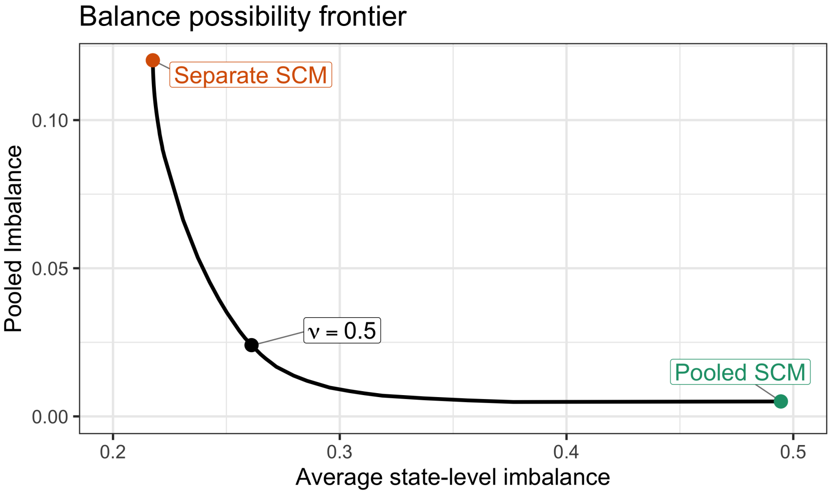


$$\min_{\Gamma} \quad \nu \, \|\text{Avg Balance}\|_2^2 + \frac{1-\nu}{J} \sum_{j=1}^{J} \|\text{State Balance}_j\|_2^2$$



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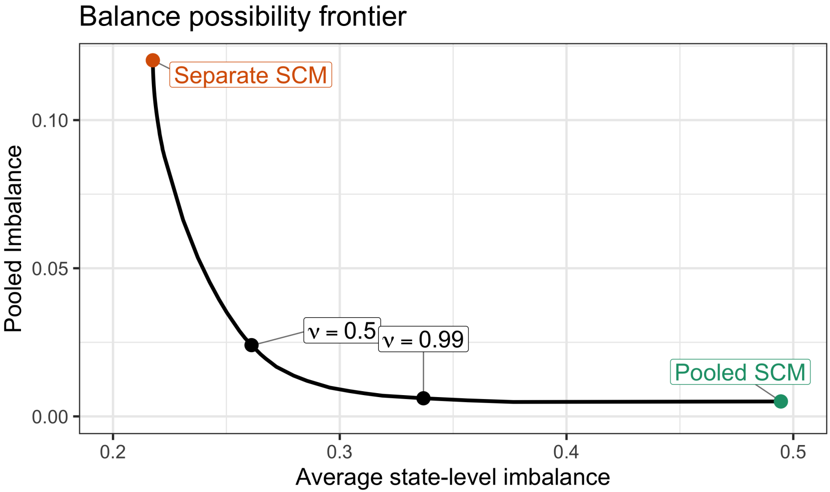
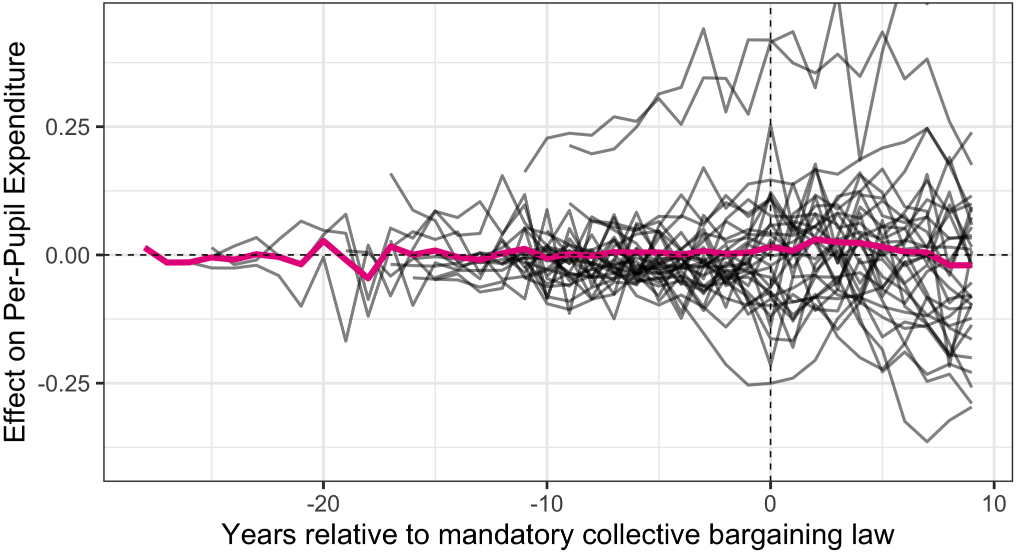


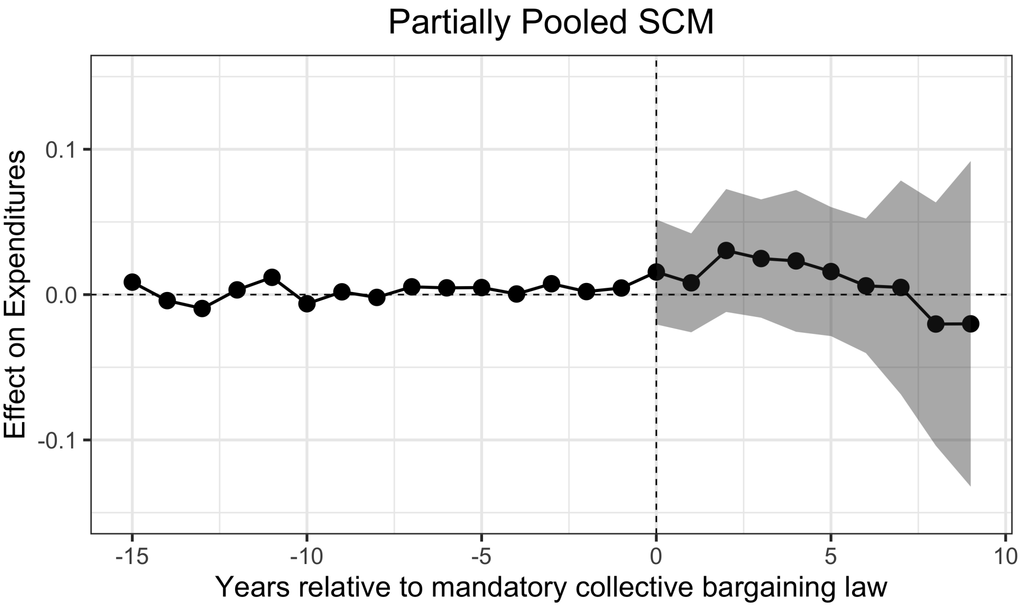
$$\min_{\Gamma} \quad \nu \left\| \text{Avg Balance} \right\|_2^2 + \frac{1-\nu}{J} \sum_{j=1}^{J} \left\| \text{State Balance}_j \right\|_2^2$$

Pooled SCM


Effect on Per-Pupil Expenditure
0.25
0.00
-0.25
-20
-10
0
10
Years relative to mandatory collective bargaining law

Partially Pooled SCM




Partially Pooled SCM
Effect on Expenditures
0.1
0.0
-0.1
-15
-10
-5
0
5
10
Years relative to mandatory collective bargaining law

Intercept Shifts

Intercept-Shifted SCM

Adjust for level differences by adding an intercept to the optimization problem

[Doudchenko and Imbens, 2017; Ferman and Pinto, 2018]

$$\hat{Y}^*_{j,T_j+k}(\infty) = \hat{\alpha}_j + \sum_i \hat{\gamma}^*_{ij} Y_{i,T_j+k}$$

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Solution: De-meaning by pre-treatment average $\bar{Y}^{\text{pre}}_{i,T_j}$

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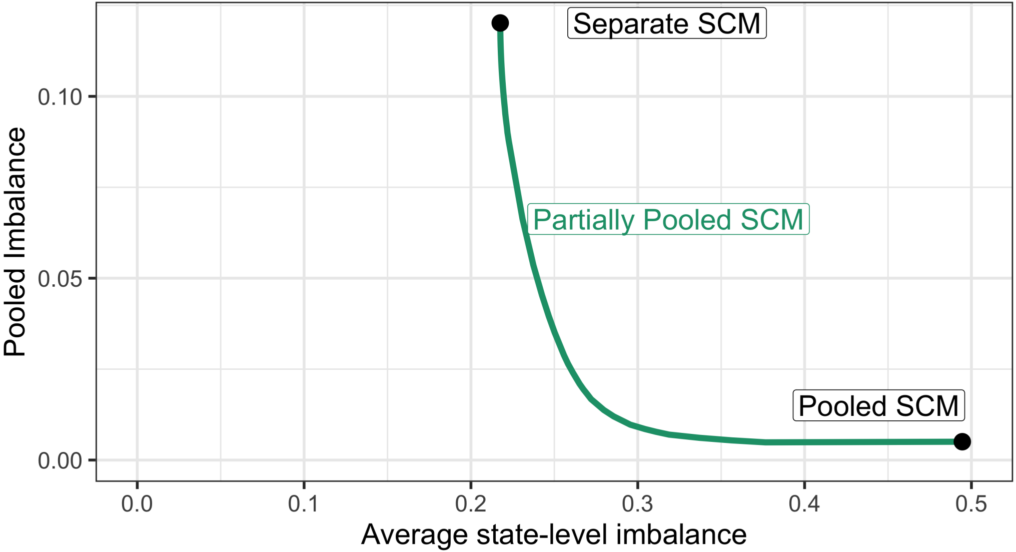
Treatment effect estimate is **weighted difference-in-differences**

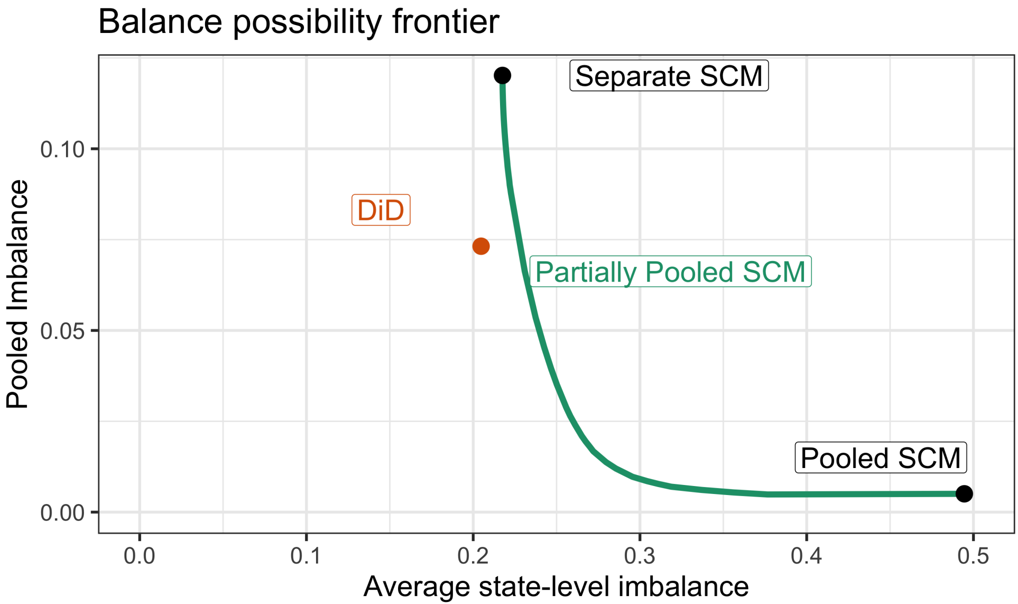
$$\hat{\tau}^{\text{aug}}_{jk} = \left(Y_{j,T_j+k} - \bar{Y}^{\text{pre}}_{j,T_j}\right) - \sum_{i=1}^{N} \hat{\gamma}^*_{ij} \left(Y_{i,T_j+k} - \bar{Y}^{\text{pre}}_{i,T_j}\right)$$

→ Uniform weights recover “stacked” DiD [Abraham and Sun, 2018]

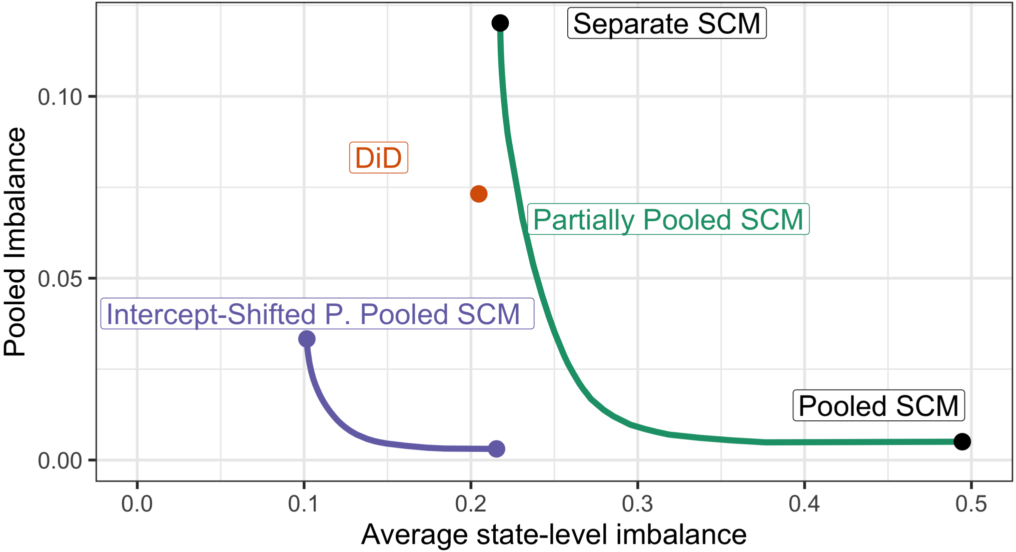
→ Similar in form to P-score weighted DiD [Abadie, 2005; Callaway and Sant’Anna, 2018]

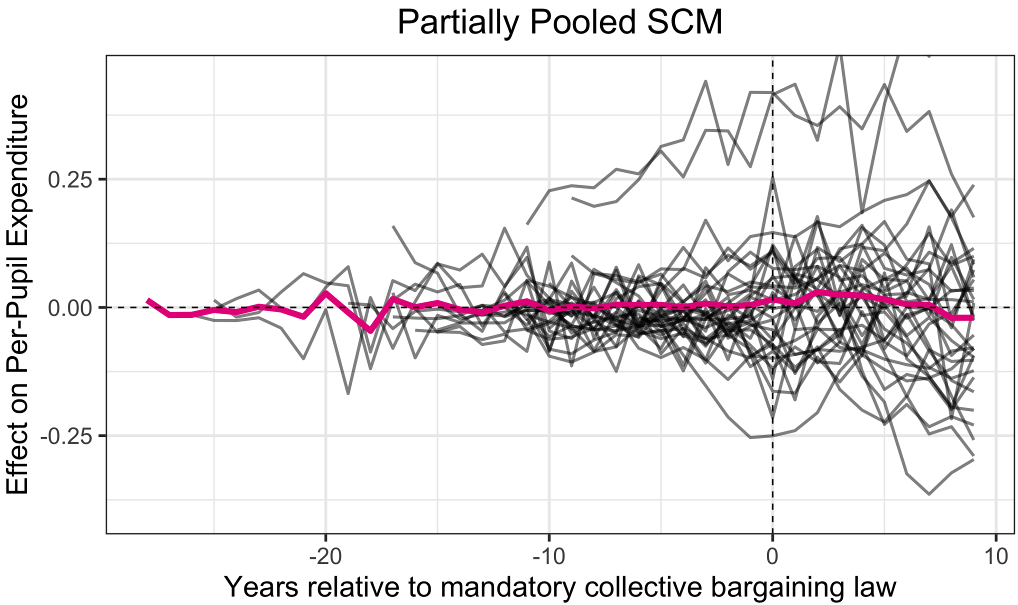
Balance possibility frontier




Balance possibility frontier
Separate SCM
DiD
Partially Pooled SCM
Pooled SCM
Pooled Imbalance
0.10
0.05
0.00
0.0
0.1
0.2
0.3
0.4
0.5
Average state-level imbalance

Balance possibility frontier





Partially Pooled SCM
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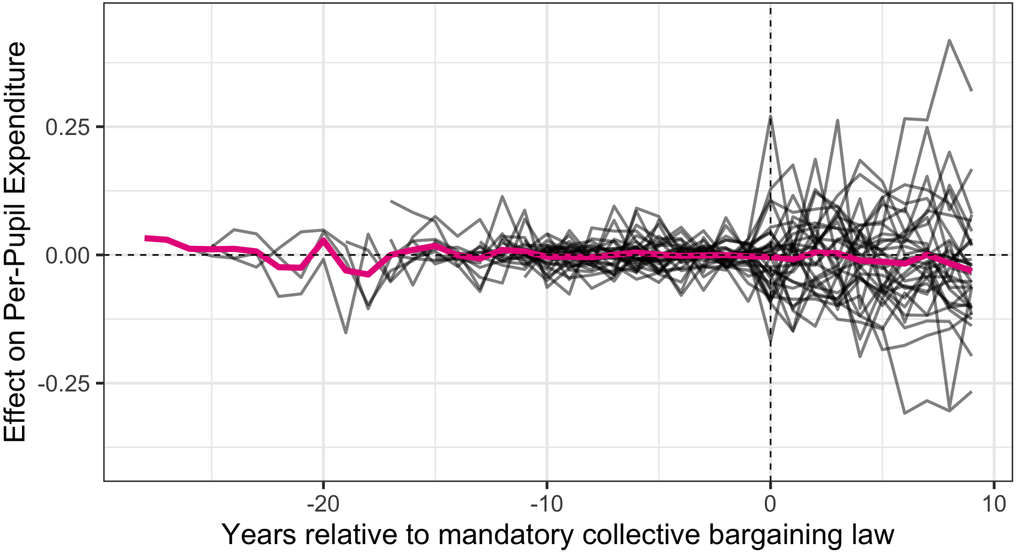
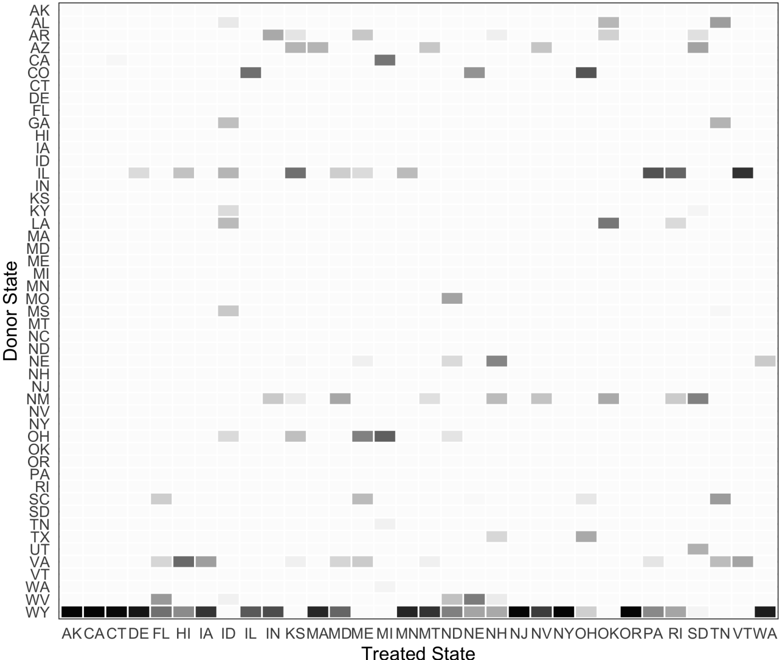
Intercept-Shifted P. Pooled SCM




Donor State
AK
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MA
MD
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OK
OR
PA
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SD
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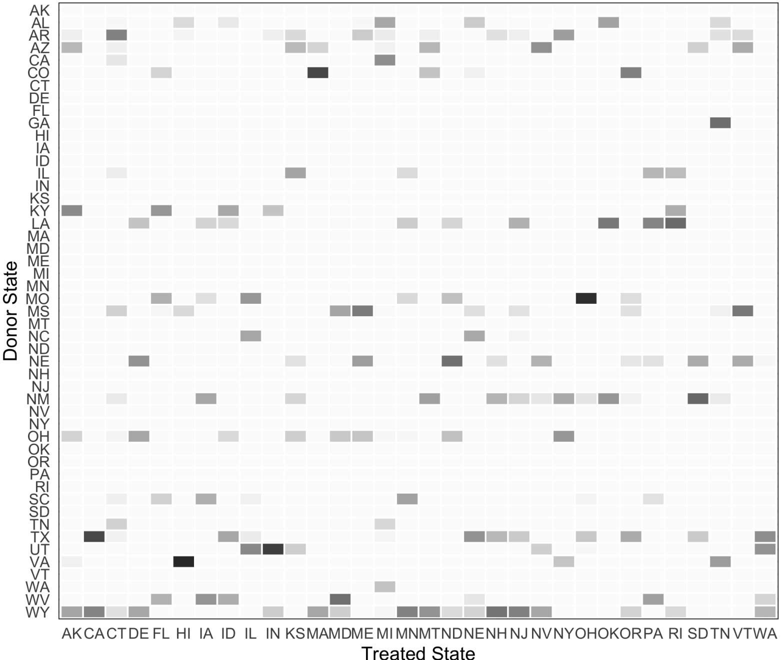
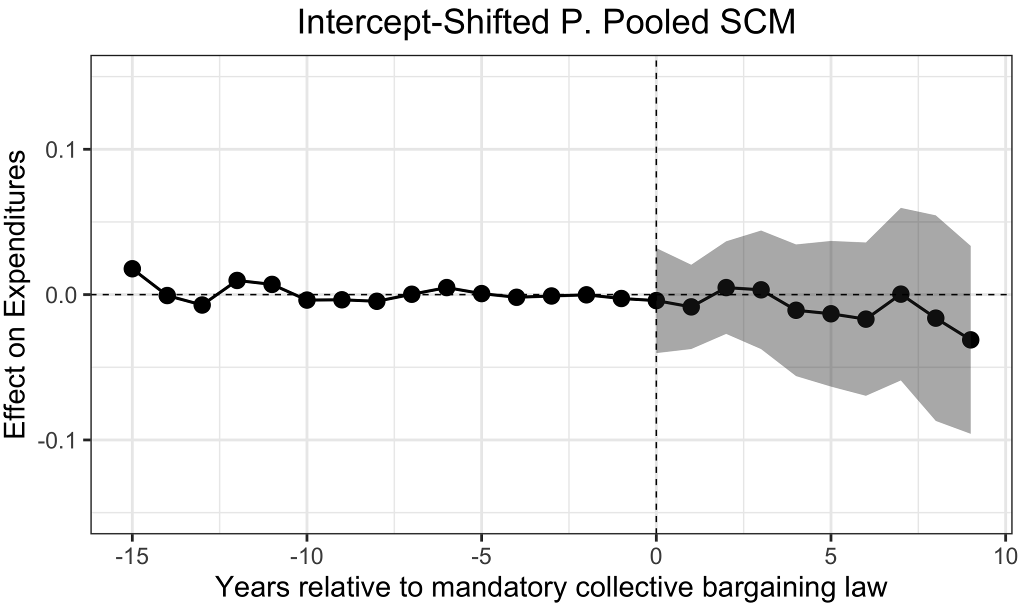
Treated State


Donor State
AK AL AR AZ CA CO CT DE FL GA HI IA ID IL IN KS KY LA MA MD ME MI MN MO MS MT NC ND NE NH NJ NM NV NY OH OK OR PA RI SC SD TN TX UT VA VT WA WV WY
Treated State
AK CA CT DE FL HI IA ID IL IN KS MA MD ME MI MN MT ND NE NH NJ NV NY OH OK OR PA RI SD TN VT WA


Intercept-Shifted P. Pooled SCM
Effect on Expenditures
0.1
0.0
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Recap

This paper: Extend SCM to staggered adoption

- Find weights that control State Balance and Avg Balance
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Extras:

- Incorporating auxiliary covariates
- Weighted bootstrap confidence intervals

In progress:

- Extend to unbalanced panels
- Sensitivity analysis

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Thank you!

https://arxiv.org/abs/1912.03290

https://github.com/ebenmichael/augsynth

Appendix

Random effects AR simulation: level of pooling really matters

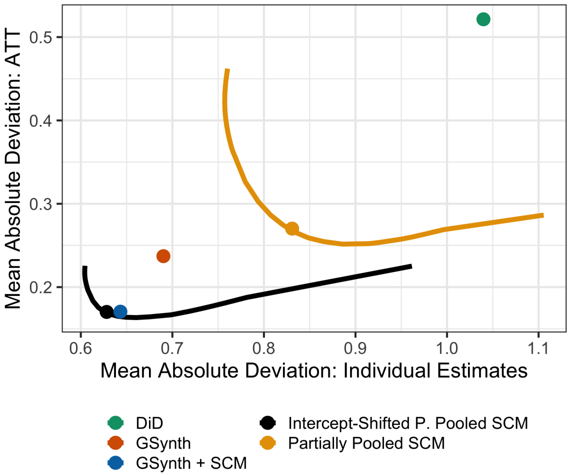
Calibrated sim study: Random Effects AR

- Fit random effects model
  [Gelman and Hill, 2007]

$$Y_{it} = \sum_{k=1}^{3} \rho_{tk} Y_{i(t-k)} + \varepsilon_{it}$$

$$\rho_t \sim N(\bar{\rho}, \Sigma)$$

- $\pi_i = \text{logit}\left(\theta_0 + \theta_1 \sum_{k=-3}^{1} Y_{i(t-k)}\right)$



DGP is FE Model: Intercept-Shifting + Partial Pooling performs well

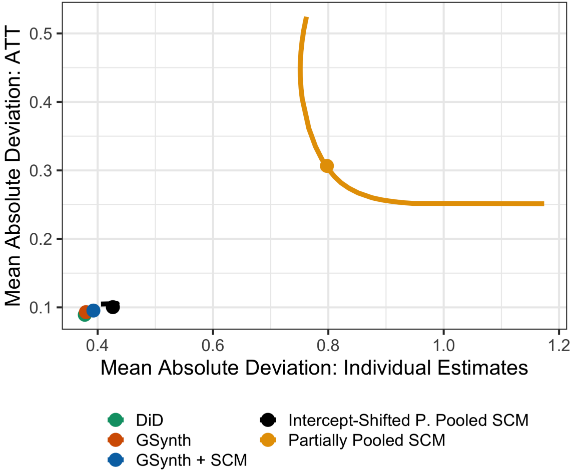
Calibrated sim study: FE

- Fit FE model

$$Y_{it} = \text{unit}_i + \text{time}_t + \varepsilon_{it}$$

- $\text{unit}_i \sim \widehat{\text{Normal}}$
- $\pi_i = \text{logit}(\theta_0 + \theta_1 \cdot \text{unit}_i)$

Event study is correct model



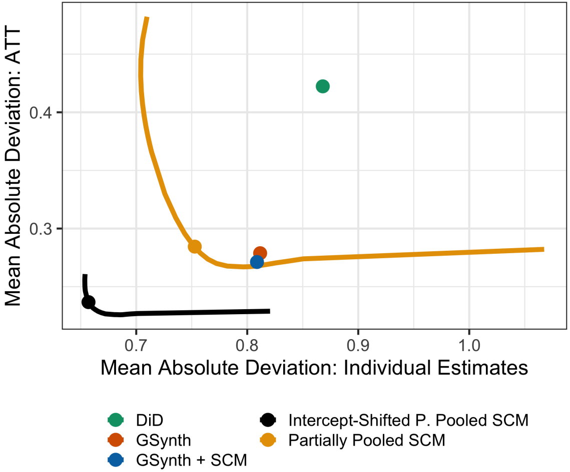
DGP is Factor Model: Intercept-Shifting + Partial Pooling does best

Calibrated sim study: Factor

- Fit gsynth [Xu, 2017]

$$Y_{it} = \text{unit}_i + \text{time}_t + \phi_i' \mu_t + \varepsilon_{it}$$

- $\{\text{unit}_i, \phi_i\} \sim \widehat{\text{MVN}}$
- $\pi_i = \text{logit}(\theta_0 + \theta_1(\text{unit}_i + \phi_{i1} + \phi_{i2}))$



Heuristic for ν: fit with $\nu = 0$ then choose

$$\hat{\nu} = \frac{\frac{1}{\sqrt{L}} \left\| \text{Avg Balance} \right\|_2}{\sqrt{\frac{1}{J} \sum_{j=1}^{J} \left\| \text{State Balance}_j \right\|_2^2}}$$

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