DiD Session Discussion, or How I Learned to Stop Worrying and Love Untestable Assumptions

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DiD/panel data: popular identification/estimation strategies

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Follow up: can we find an estimator that estimates what we want, under the assumptions that we're willing to make?

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For a 2x2 DiD

$$\hat{\tau}_{11}^{\text{DiD}} = \mathbb{E}[Y_{\text{post}}(1) - Y_{\text{post}}(0) \mid G = 1] - \mathbb{E}[Y_{\text{post}}(1) - Y_{\text{post}}(0) \mid G = 0] \quad \text{(difference in post ATTs)}$$
$$- (\mathbb{E}[Y_{\text{pre}}(1) - Y_{\text{pre}}(0) \mid G = 1] - \mathbb{E}[Y_{\text{pre}}(1) - Y_{\text{pre}}(0) \mid G = 0]) \quad \text{(difference in pre ATTs)}$$

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$$\hat{\tau}_{11}^{\text{DiD}} = \mathbb{E}[Y_{\text{post}}(1) - Y_{\text{post}}(0) \mid G = 1]$$

XZD: No anticipation, both groups treated $\Rightarrow$ pre-ATT is 0 for both groups

- insight: useful to define add'l layer of POs based on group $Y_t(g, z)$

$$\begin{aligned}\hat{\tau}_{11}^{\text{DiD}} &= \mathbb{E}[Y_{\text{post}}(1) - Y_{\text{post}}(0) \mid G = 1] - \mathbb{E}[Y_{\text{post}}(1) - Y_{\text{post}}(0) \mid G = 0] \\ &= E[Y_{\text{post}}(1,1) - Y_{\text{post}}(1,0) \mid G = 1] - \mathbb{E}[Y_{\text{post}}(0,1) - Y_{\text{post}}(0,0) \mid G = 0]\end{aligned}$$

What do we want to estimate?

LS: In typical DiD setting, want a notion of average treatment effect on the treated

$$\text{effect} = \sum_{\text{times } t} \sum_{\text{ever trt groups } g} \text{weight}_{gt} \times \text{ATT}_g(t)$$

- For some aggregation strategy, what would be the average difference in outcomes under treatment and control across all groups and times?

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XZD: The **causal interaction** between group status and treatment

$$\text{interaction effect} = \mathbb{E}[(Y_{\text{post}}(1,1) - Y_{\text{post}}(1,0)) - (Y_{\text{post}}(0,1) - Y_{\text{post}}(0,0))]$$

- What would be the average difference in effects, if everyone was in group 1 vs. group 0?

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XZD: Under generalized DiD, the 2x2 DiD estimator is estimating a weird thing that is really just a description of heterogeneity

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  - Same relative time effects across groups (Assumption 6)

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XZD: Restrictions on heterogeneity

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- Parallel trends across both levels of both treatments ($2 \times 2 = 4$ sets of constraints)
- What this buys us: conditioning on group doesn't matter, and so description = causal

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- Is this assumption palatable?
  - Possible diagnostic with multiple periods and add'l (strong!) assumption of no effect carryover

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XZD: Unclear

- Are there opportunities to use multiple periods?
- Are we just out of luck?

XZD: Exclusion restriction and cannonical DiD

GDID setting recovers typical DiD setting under **exclusion restriction** (Assumption 5)

$$Y_{\text{post}}(0,1) = Y_{\text{post}}(0,0)$$

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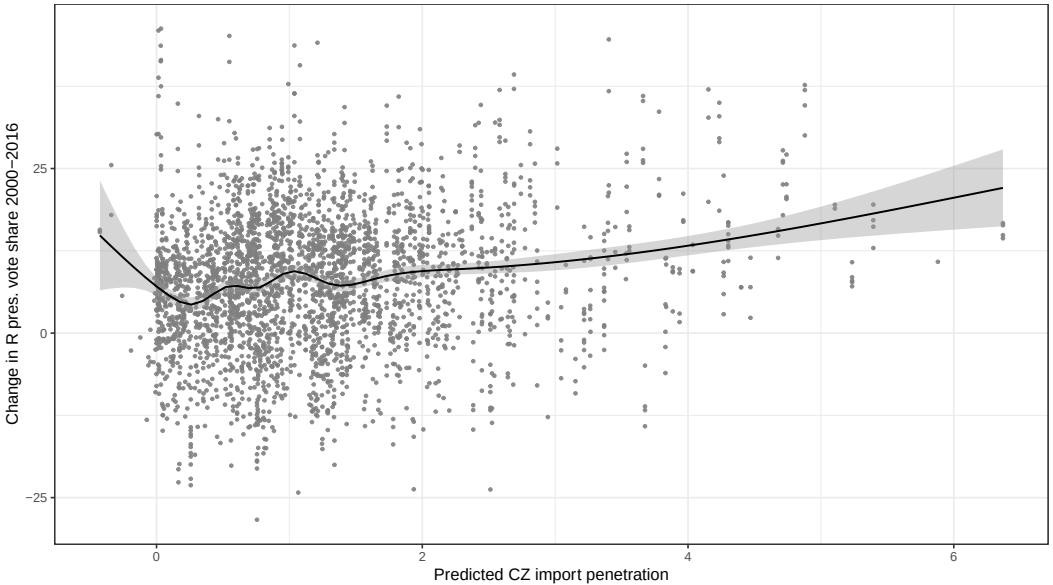
Are Bartik instruments/shift-share designs the other side of the coin?

- Groups defined by G have different level of exposure to Z
- e.g. the "China shock" [Autor et al., 2013, 2020]
- IV-style identification, but also fixed-effects/first differencing
  [Goldsmith-Pinkham et al., 2020; Borusyak et al., 2022; Borusyak and Hull, 2023]
- Focus is on estimating effect of Z, not interaction, but does the GDID-style of analysis have something to say about shift-share designs, and vice versa?

Heterogeneity in impacts of “China shock” on presidential elections?



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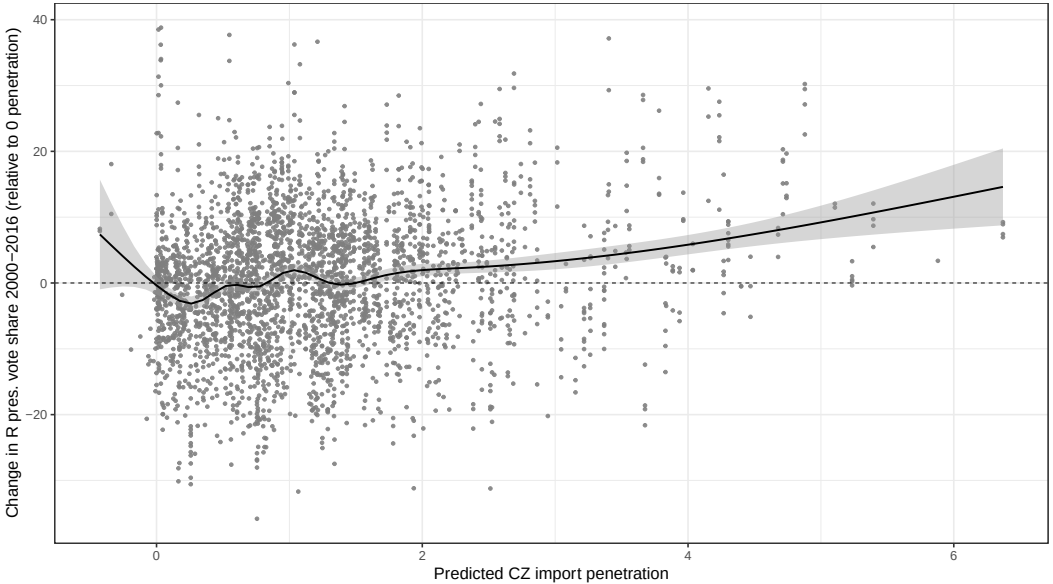

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To what extent are these issues are a matter of degree?

- Discussion about omitted relative time period
  - "Omitting a baseline period that does not appear for all units... places a much greater weight on the effect homogeneity assumption" (pg 28)
  - "Omitting pre-treatment relative time indicators that are too far from the start of treatment risks placing greater weight on the effect homogeneity assumption" (pg 35)

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Including never-treated units vs sometimes-treated units

- Paglayan and Hall & Yoder examples: evidence of pre-trends with sometimes-treated units
- Q in paper: does including never treated units make it *worse*?
- Regardless, implies we shouldn't include sometimes-treated units

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Strong form of parallel trends $\Rightarrow$ need to check for pre-trends across all pairs of groups?

- Omnibus diagnostic for strong parallel trends?
- Removing comparison pairs w/o parallel trends?
- But also pre-testing in problematic [Roth, 2018]

Your turn: Q&A

References I

Autor, D., Dorn, D., Hanson, G., and Majlesi, K. (2020). Importing Political Polarization? The Electoral Consequences of Rising Trade Exposure. *American Economic Review*, 110(10):3139–3183.

Autor, D. H., Dorn, D., and Hanson, G. H. (2013). The China Syndrome: Local Labor Market Effects of Import Competition in the United States. *American Economic Review*, 103(6):2121–2168.

Borusyak, K. and Hull, P. (2023). Nonrandom Exposure to Exogenous Shocks. *Econometrica*, 91(6):2155–2185. _eprint: https://onlinelibrary.wiley.com/doi/pdf/10.3982/ECTA19367.

Borusyak, K., Hull, P., and Jaravel, X. (2022). Quasi-Experimental Shift-Share Research Designs. *The Review of Economic Studies*, 89(1):181–213.

Goldsmith-Pinkham, P., Sorkin, I., and Swift, H. (2020). Bartik Instruments: What, When, Why, and How. *American Economic Review*, 110(8):2586–2624.

Roth, J. (2018). Should we condition on the test for pre-trends in difference-in-difference designs? *arXiv preprint arXiv:1804.01208*.