Think Globally, Balance Locally:

Multi-Level Balancing Weights for

Multi-Site Observational Studies

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(joint work with Avi Feller)

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Typical Inverse Propensity Weighting (IPW) workflow

Probability
of treatment

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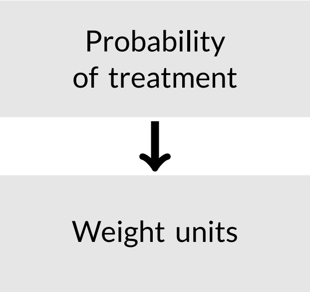

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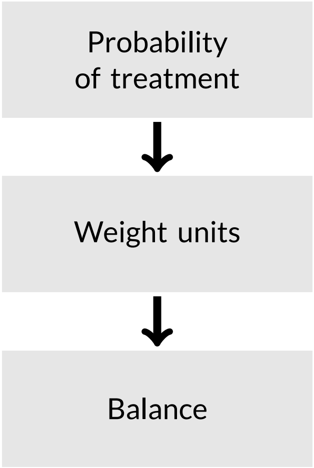

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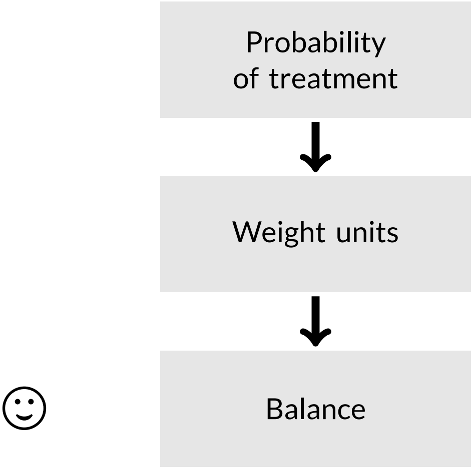

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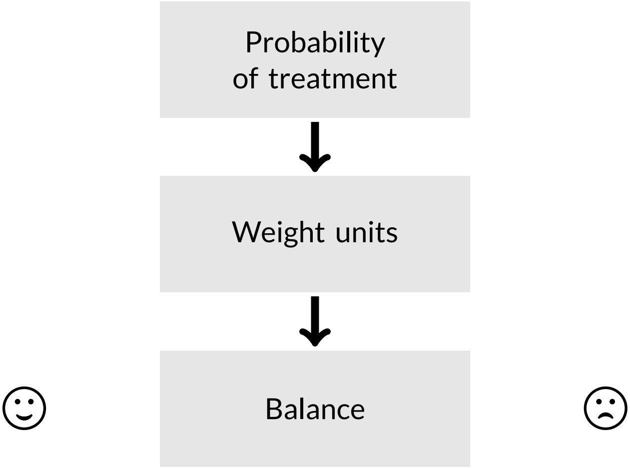
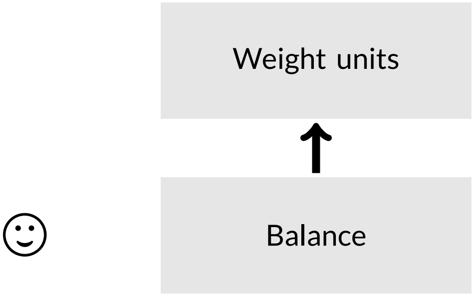


Balancing weights workflow

☺ Balance

Balancing weights workflow



Balancing weights workflow

Probability of treatment

↑

Weight units

↑

☺ Balance

Multi-level balancing weights

This talk: Extend balancing weights to multi-level setting

Multi-level balancing weights

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Multilevel Matching [Zubizarreta and Keele, 2017; Pimentel et al., 2018]	—	Hierarchical IPW [Arpino and Mealli, 2011; Li et al., 2013]

Multi-level balancing weights

This talk: Extend balancing weights to multi-level setting

Multilevel Matching

[Zubizarreta and Keele, 2017; Pimentel et al., 2018]

Hierarchical IPW

[Arpino and Mealli, 2011; Li et al., 2013]

Multilevel Weighting

Multi-level selection mechanisms

Cluster-level treatment assignment

[Zubizarreta and Keele, 2017; Pimentel et al., 2018]

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Multi-site studies

- **Ex**: Obs. study simulation from real RCT for ACIC workshop

  Based off of National Study of Learning Mindsets [Yeager, 2017]

The balance objective depends on the estimand

Two balancing goals:

Overall effect	$\longleftrightarrow$	Balance globally across all schools
School-level effects	$\longleftrightarrow$	Balance locally within each school

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Two balancing goals:

Overall effect $\longleftrightarrow$ Balance globally across all schools

School-level effects $\longleftrightarrow$ Balance locally within each school

Really want to do both:

Balancing within school and globally

$\updownarrow$

Partial pooling in multilevel IPW

What we see

For student i in school $j[i]$ observe:

- Student-level covariates $X_i \in \mathbb{R}^d$ and school-level covariates $V_{j[i]} \in \mathbb{R}^p$
- Treatment status T_i, School indicator S_i
- Outcome: $Y_i = Y_i(1)T_i + Y_i(0)(1 - T_i)$

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Goal: Estimate the ATT

$$\tau = \mathbb{E}[Y(1) - Y(0) \mid T = 1]$$

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Goal: Estimate the ATT **and school CATT**

$$\tau = \mathbb{E}[Y(1) - Y(0) \mid T = 1]$$

$$\tau_j = \mathbb{E}[Y(1) - Y(0) \mid T = 1, S = j]$$

What we assume

Key Identifying assumption: Strong ignorability

$$Y(1), Y(0) \perp T \mid X, V \quad \text{and} \quad \pi(X, V) \equiv P(T = 1 \mid X, V) < 1$$

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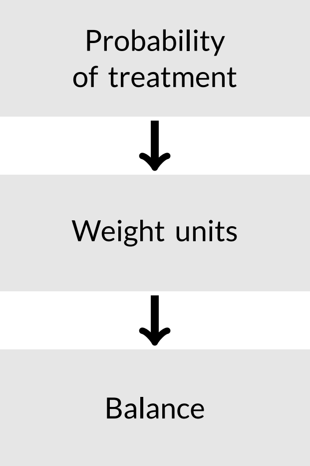
Weighting control units by the odds of treatment:

$$\mathbb{E}[Y(0) \mid T = 1] = \mathbb{E}\left[\frac{\pi(X, V)}{1 - \pi(X, V)} Y \mid T = 0\right]$$

Two views of the P-score imply different estimation methods

Conditional probability of treatment

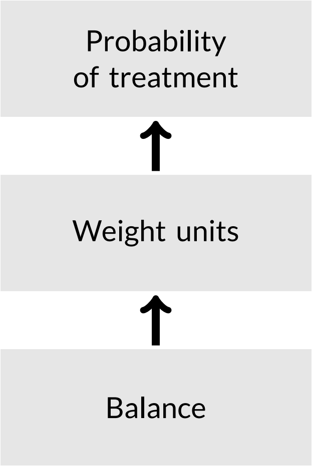
- Estimate $\hat{\pi}(X_i, V_i)$ with MLE, then estimate weights $\hat{\gamma}_i = \frac{\hat{\pi}(X_i, V_{j[i]})}{1-\hat{\pi}(X_i, V_{j[i]})}$
- Indirectly balances covariates
- Poor finite sample performance, especially with many covariates
  [Athey et al., 2018]



Two views of the P-score imply different estimation methods

Balancing score [Rosenbaum and Rubin, 1983]

- Find weights $\hat{\gamma}$ that directly balance covariates
- Indirectly estimates the P-score
- Old history as raking and calibration in survey sampling with non-response
  [Deming and Stephan, 1940; Deville et al., 1993]



Balancing globally estimates the overall propensity score

Balancing weights:

[Hainmueller, 2011; Zubizarreta, 2015]

Global Balance $= 0$

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Calibrated propensity score:

[Tan, 2017; Wang and Zubizarreta, 2018; Zhao, 2017]

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Linking the two: [Zhao and Percival, 2016]

$$\hat{\gamma}_i = \exp(\hat{\alpha} + {\hat{\mu}_\beta}' X_i + \hat{\eta}' V_{j[i]}) = \frac{\hat{\pi}(X_i, V_{j[i]})}{1 - \hat{\pi}(X_i, V_{j[i]})}$$

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Calibrated propensity score: Complete Pooling
[Tan, 2017; Wang and Zubizarreta, 2018; Zhao, 2017]

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Balancing each school fits separate propensity scores

With multiple sites, can restrict to within school analysis, follow same procedure

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$$\beta_j \sim N(0, \sigma_\beta^2)$$

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Within school balance $\not\Rightarrow$ global balance because it is only *approximate*

*Technically regularization

Balancing both across and within schools is partial pooling

Measuring balance:

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$$+$$

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Balancing both across and within schools is partial pooling

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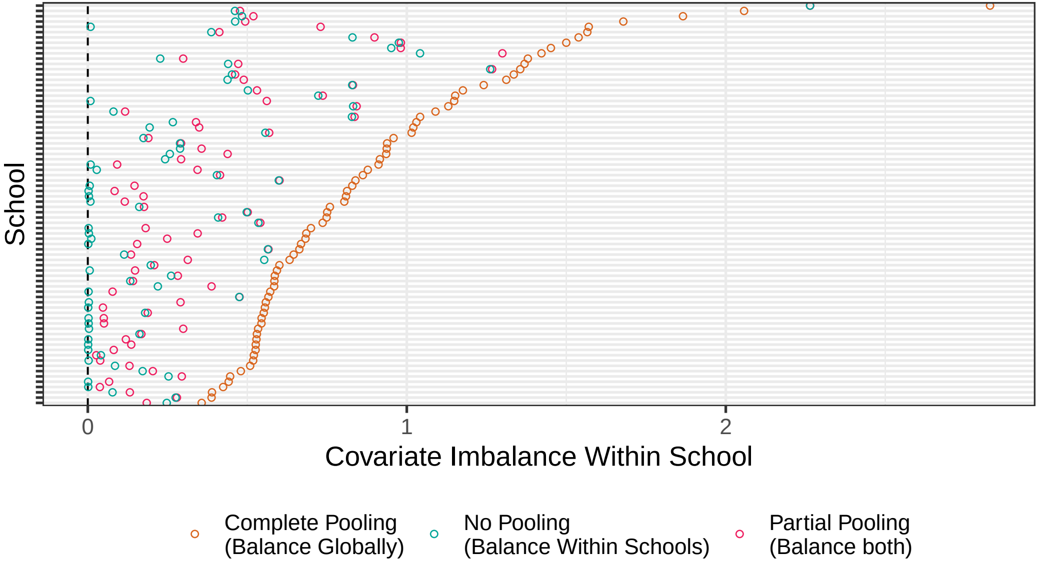
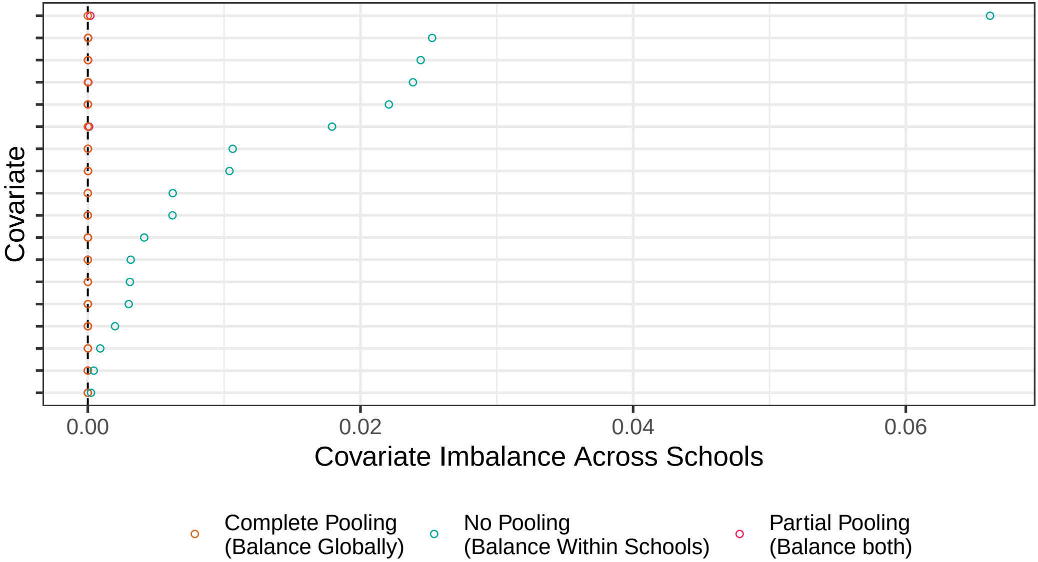
Hierarchical propensity score: (FIRC)

$$\begin{aligned}\pi(X_i, V_{j[i]}) &= \text{logit}^{-1}(\alpha_j + \beta_j{}' X_i)\\ \beta_j &\sim N\left(\mu_\beta + \eta' V_j, \sigma_\beta^2\right)\end{aligned}$$

Partial pooling achieves good balance within school



Partial pooling achieves nearly perfect balance across schools



Summary

In multi-site observational studies we want to balance:

- Globally across schools
- Locally within schools

But just balancing one does not balance the other

Solution: balance both

- Implicitly fits a multi-level propensity score model (fixed intercept random coefficients)

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Thanks!

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`ebenmichael.github.io`

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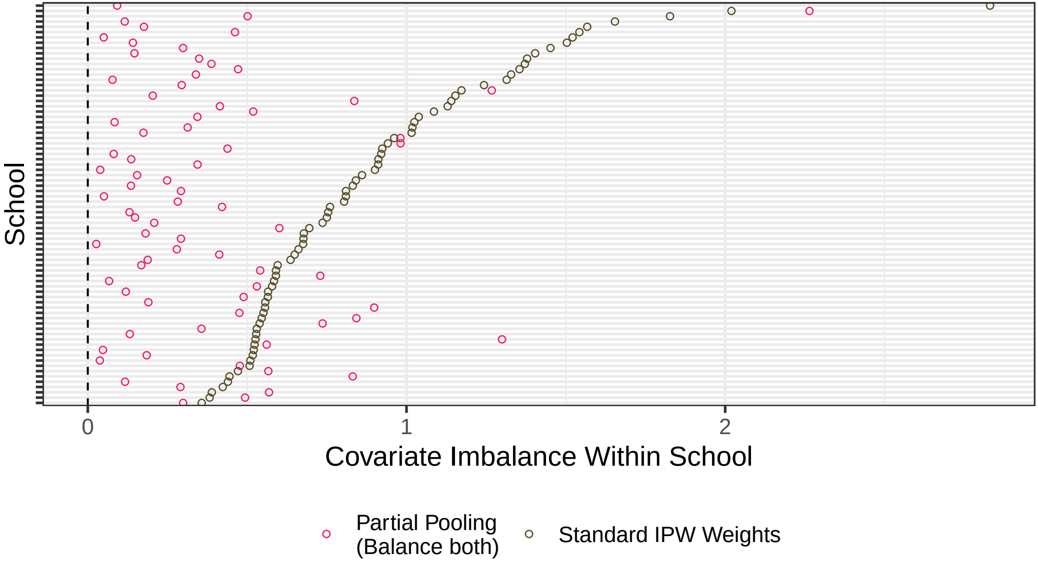
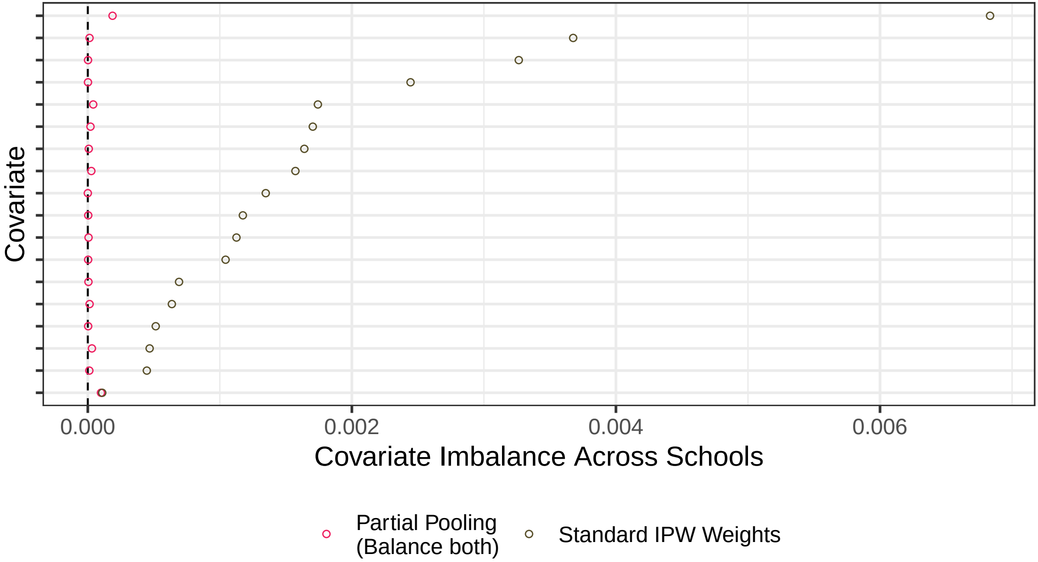
Appendix

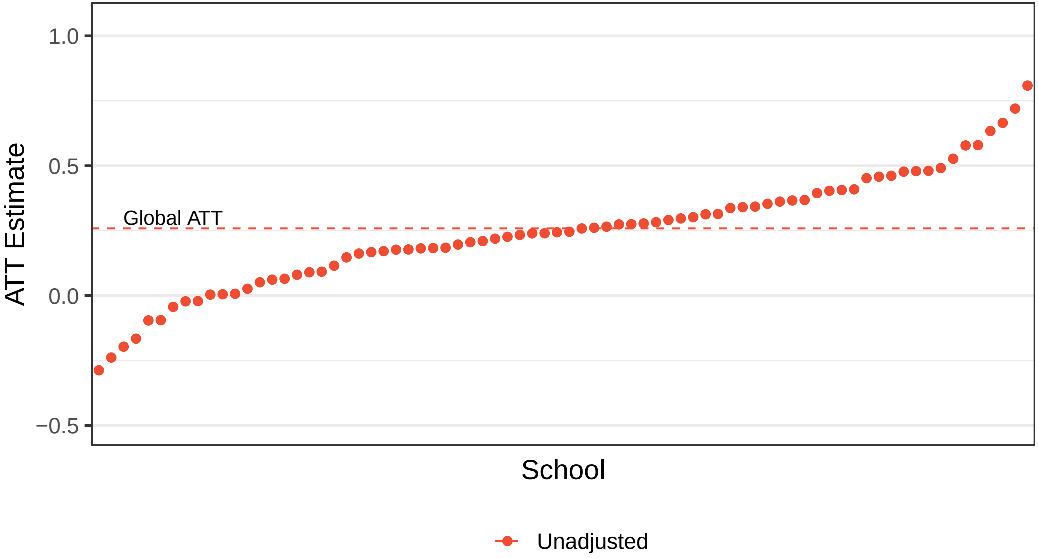
Standard multilevel IPW achieves worse balance within school



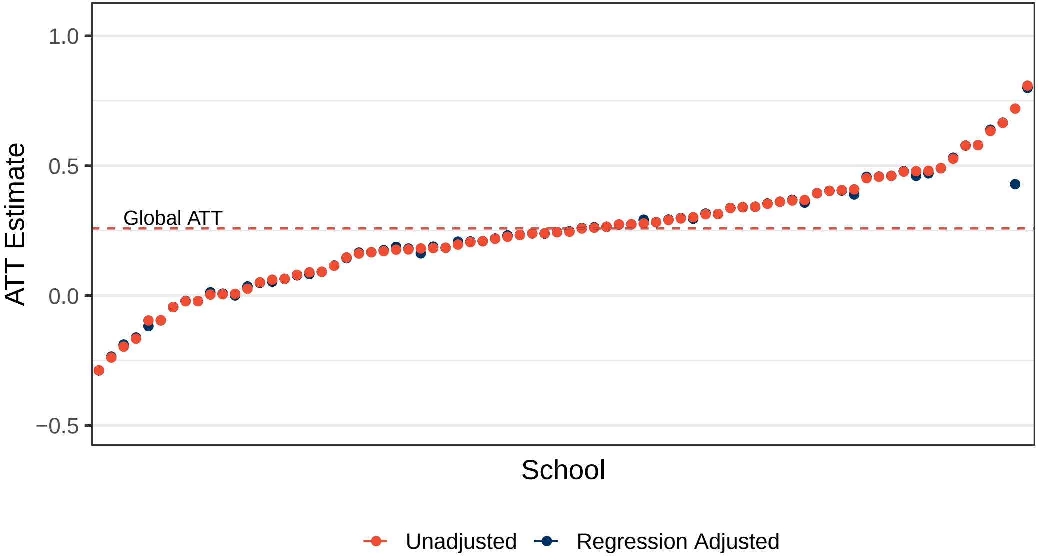
Standard multilevel IPW achieves worse balance globally



Estimating overall and school-specific effects



Bias Correction



Pooling ATT Estimates